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Stochastic synchronization for an array of hybrid neural networks with random coupling strengths and unbounded distributed delays☆

Cheng-De Zheng[a,*], Qihe Shan[b], Ziping Wei[a]

[a] School of Science, Dalian Jiaotong University, Dalian 116028, PR China
[b] Navigation College, Dalian Maritime University, Dalian 116026, PR China

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ABSTRACT

This paper investigates the globally asymptotic synchronization for hybrid neural networks with random coupling strengths and mixed time-delays in the mean square. By employing a novel augmented Lyapunov–Krasovskii functional (LKF), applying the theory of Kronecker product of matrices, Barbalat's Lemma and the auxiliary function-based integral inequalities, several novel delay-dependent conditions are established to achieve the globally stochastic synchronization for the hybrid coupled neural networks. Two presented criteria do not require all the symmetric matrices involved in the employed quadratic LKF to be positive definite. Furthermore, the conservatism of delay-dependent stability conditions can be reduced due to the relaxation on the positive-definiteness of some Lyapunov matrices. Finally, two numerical examples with simulation are provided to illustrate the effectiveness of the presented criteria.

1. Introduction

During the last few years, complex networks have received increasing attention from the real world such as the internet, social networks, electrical power grids, global economic markets, and so on. Many interesting behaviors can be observed from complex dynamical networks, e.g., synchronization, consensus, self-organization, and spatiotemporal chaos spiral waves. As an important collective behavior of complex dynamical networks, synchronization has been widely investigated in the last two decades (see, for example, [30,34]).

As a special class of complex networks, coupled neural networks have been a hot topic because they have wide applications in a variety of areas, such as signal processing, pattern recognition, static image processing, associative memory, and combinatorial optimization [3,11,16,25,28]. On the other hand, due to the finite speeds of the switching and transmitting signals, time delays exist in neural networks. It is well known that time delays may result in oscillatory behaviors or network instability (periodic oscillation and chaos). So far, most of the existing results related to the synchronization analysis for neural networks have been concerned with the discrete time-delay. The distributed time-delay has received increasing research interest due to the presence of an amount of parallel pathways with a variety of axon sizes and lengths [5,14]. In [2], based on LKF method and Kronecker product techniques, several synchronization criteria have been obtained for identical neural networks with constant and delayed coupling by means of linear matrix inequalities (LMIs). In [1], by introducing several new free-weighting matrices and using the Jensen integral inequality, several LMI-based synchronization criteria have been established for coupled neural networks with constant coupling, discrete-delay coupling and distributed-delay coupling. In [15], by utilizing a novel LKF and Barbalat's Lemma, a few LMI-based synchronization criteria have been achieved for linearly coupled neural networks with discrete and unbounded distributed time-delays. In [33], by introducing several new free-weighting matrices and using the Jensen inequality, two LMI-based synchronization criteria have been acquired for neutral-type neural networks with mixed time delays and hybrid nonlinear coupling strengths. In [18], by devising a new zero equality, utilizing Finsler's lemma and the Jensen inequality, two LMI-based synchronization criteria have been derived for coupled neural networks with interval time-varying delays in network coupling and leakage delay. In [13], a few LMI-based synchronization conditions have been proposed for identical neutral-type Markovian coupled neural networks with mode-dependent discrete and unbounded distributed time delays by introducing a novel LKF and using some analytical techniques. In [24], by constructing an appropriate augmented LKF, introducing several new free-weighting matrices and using the Jensen inequality, an LMI-based synchronization condition has been presented for

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* Corresponding author.
E-mail addresses: chd4211853@163.com, zhengchengde@dl.cn (C.-D. Zheng), smilingtear007@163.com (Q. Shan).

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nonlinear coupled static neural networks with time-varying delay. In [4], by introducing several new free-weighting matrices, using Barbalat's Lemma and the Jensen inequality, two LMI-based synchronization conditions have been introduced for hybrid coupled neural networks with leakage delay, time-varying discrete and distributed delays. In [8], based on Kronecker product techniques and the Jensen inequality, two LMI-based synchronization criteria have been reported for static neural networks with hybrid couplings and constant time delays.

On the other hand, it is well known that coupling strength plays an important role while studying synchronization of complex networks. If coupling strength is sufficiently large, a complex network can synchronize itself or can be pinned to a desired objective state (see [29]). To the best of our knowledge, few published papers considered random coupling strength while studying dynamics of complex networks. Obviously, when the coupling strength varies randomly, conditions for synchronization of complex networks with constant coupling strength may be inapplicable anymore, or more conservative result would be derived if only the upper or lower bound of the random coupling strength is utilized. Therefore, it is urgent to investigate the synchronization of complex networks with random coupling strength, which motivates us to study the synchronization problem for hybrid coupled neural networks with time-varying discrete delay and unbounded distributed delays.

As is well known, in the field of synchronization analysis, obtaining tight bounds of integral terms of quadratic functions plays a key role in reducing the conservatism. In the last decade, the Jensen inequality has been intensively used to acquire these bounds. Recently, Seuret and Gouaisbaut [21] proposed a Wirtinger-based integral inequality, which includes the Jensen one as a special case and yields tighter bounds of integral terms of quadratic functions than the Jensen one. In order to further reduce conservatism from the use of the Jensen inequality and the Wirtinger-based one, Park et al. [20] developed some auxiliary function-based integral inequalities, which encompass the Wirtinger-based one and yields tighter bounds of integral terms of quadratic functions than the Wirtinger-based one. Besides a new established inequality in Lemma 7, we will utilize some auxiliary function-based integral inequalities to get our main results.

This paper investigates the stochastic synchronization problem for a class of hybrid neural networks with random coupling strengths and mixed time-delays in the mean square. The main contributions of this paper can be summarized as follows:

(1) Not all the symmetric matrices involved in the employed quadratic LKF are required to be positive definite, for detail please refer to Remark 3.
(2) A novel inequality (see Lemma 7) is established, which is a double integral form of the Wirtinger-based integral inequality [21], includes the Jensen one as a special case and yields tighter bounds of integral terms of quadratic functions than the Jensen one. Based on this new inequality, we can utilize more information to obtain less conservative delay-dependent synchronization conditions.
(3) By applying a simple variation of the reciprocal convex approach in [17], tighter upper bounds of some reciprocal convex combinations are obtained with less conservative approximation. For detail please refer to Remark 6.
(4) Proper combination of the novel inequality (see Lemma 7) and some auxiliary function-based integral inequalities (see Lemmas 5, 6, 8) with the reciprocal convex combination technique assures us to derive less conservative synchronization criteria.

Notations: Throughout this paper, let W^T, W^{-1} denote the transpose and the inverse of a square matrix W, respectively. $W>0(<0)$ denotes a positive (negative) definite symmetric matrix, I denotes the identity matrix with compatible dimension, $0_{m\times n}$ denotes the $m\times n$ zero matrix, the symbol "$*$" denotes a block that is readily inferred by symmetry. The shorthand $\text{col}\{M_1, M_2, \dots, M_k\}$ denotes a column matrix with the matrices $M_1, M_2, \dots, M_k$. $\text{sym}(A)$ is defined as $A+A^T$, $\text{diag}\{\cdot\}$ stands for a diagonal or block-diagonal matrix. $\mathbb{E}\{\cdot\}$ represents the mathematical expectation. $||\cdot||$ stands for the Euclidean norm; matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. Problem description and preliminaries

In this paper, we consider the following hybrid neural networks with mixed delays and random coupling strengths:

$$\begin{aligned}\dot{x}_i(t) = &-Dx_i(t) + Af(x_i(t)) + Bf(x_i(t-\tau(t)))\\ &+C\int_{-\infty}^{t-\sigma}\kappa(t-s)f(x_i(s))\mathrm{d}s\\ &+\alpha(t)\sum_{j=1}^{m}u_{ij}\Phi x_j(t) + \beta(t)\sum_{j=1}^{m}v_{ij}\Psi x_j(t-\tau(t))\\ &+\gamma(t)\sum_{j=1}^{m}w_{ij}\Upsilon\int_{-\infty}^{t-\sigma}\kappa(t-s)x_j(s)\mathrm{d}s + \psi(t),\\ &\qquad i\in M=\{1,2,\dots,m\}. \end{aligned} \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \dots, x_{in}(t))^T \in \mathbb{R}^n$ is the state vector of the ith node of the coupled networks at moment t, positive integer n corresponds to the number of neurons. $D = \text{diag}\{d_1, d_2, \dots, d_n\}$ is a positive diagonal matrix with d_j representing the rate with which the jth neuron will reset its potential to the resting state in isolation, A, B, C are the connection weight matrix, the discretely delayed connection weight matrix, the distributively delayed connection weight matrix, respectively. $f(x_i(t)) = \big(f_1(x_{i1}(t)), f_2(x_{i2}(t)), \dots, f_n(x_{in}(t))\big)^T \in \mathbb{R}^n$ denotes the neural activation function. Positive scalar σ, bounded function $\tau(t)$ represent unknown discrete and time-varying delays respectively with $0\le\tau(t)\le\bar{\tau}, \dot{\tau}(t)\le\tau'$, where $\bar{\tau}$ is a positive scalar. $\kappa(\cdot):[0,+\infty)\to[0,+\infty)$ is the delay kernel. Random variables $\alpha(t)$, $\beta(t)$ and $\gamma(t)$ are mutually independent, which denote the random coupling strengths of non-delayed couplings, discretely time-delayed couplings and distributively time-delayed couplings respectively. We assume that almost all the values of $\alpha(t)$, $\beta(t)$ and $\gamma(t)$ are taken on some nonnegative intervals, i.e., $\alpha(t)\in(\rho_1(t),\ \eta_1(t))$, $\beta(t)\in(\rho_2(t),\ \eta_2(t))$ and $\gamma(t)\in(\rho_3(t),\ \eta_3(t))$, where $\rho_j(t)$, $\eta_j(t)$ are nonnegative constants with $\rho_j(t)<\eta_j(t)$, $j=1,2,3$. $\psi(t)$ is an external input vector, Φ, Ψ and Υ are inner coupling matrices between coupled nodes. Symmetric matrices $U=(u_{ij})_{m\times m}, V=(v_{ij})_{m\times m}$ and $W=(w_{ij})_{m\times m}$ stand for outer coupling matrices of the whole network satisfying the following diffusive conditions: $u_{ij}\ge 0(i\ne j),\ u_{jj}=-\sum_{i=1,i\ne j}^{m}u_{ij};\ v_{ij}\ge 0(i\ne j),\ v_{jj}=-\sum_{i=1,i\ne j}^{m}v_{ij};\ w_{ij}\ge 0(i\ne j),\ w_{jj}=-\sum_{i=1,i\ne j}^{m}w_{ij},\ i,j\in M.$

The initial value of networks (1) is given by $x_i(s)=\varphi_i(s)$, where $\varphi_i(s)$ is a continuous function from $(-\infty,0]$ to $\mathbb{R}^n$.

Throughout this paper, we make the following assumptions and definition.

Assumption 1 (Liu et al. [14]). There exist constants l_j^-, l_j^+ such that

$$l_j^- \le \frac{f_j(p)-f_j(q)}{p-q} \le l_j^+, \qquad j\in N=\{1,2,\dots,n\},$$

for any $p,q\in\mathbb{R}$ with $p\ne q$.

For simplicity, we denote $L_1=\text{diag}\{l_1^-l_1^+, l_2^-l_2^+, \dots, l_n^-l_n^+\}$, $L_2=\frac{1}{2}\text{diag}\{l_1^-+l_1^+, l_2^-+l_2^+, \dots, l_n^-+l_n^+\}$.