Neurocomputing 000 (2017) 1–7

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Further results on L_2–L_∞ state estimation of delayed neural networks

Wei Qian [a,*], Yonggang Chen [b,c], Yurong Liu [d], Fuad E. Alsaadi [e]

[a] College of Electrical Engineering and Automation, Henan Polytechnic University, Jiaozuo 454000, PR China
[b] School of Mathematical Sciences, Henan Institute of Science and Technology, Xinxiang 453003, PR China
[c] Key Laboratory of Control Engineering of Henan Province, Henan Polytechnic University, Jiaozuo 454000, PR China
[d] Department of Mathematics, Yangzhou University, Yangzhou 225002, PR China
[e] Faculty of Engineering, King Abdulaziz University, Jeddah 21589, Saudi Arabia

ARTICLE INFO

Article history:
Received 7 May 2017
Revised 8 August 2017
Accepted 14 August 2017
Available online xxx

Communicated by Prof. Wei Guoliang

Keywords:
State estimation
Neural networks
Time-varying delay
L_2–L_∞ performance

ABSTRACT

This paper investigates the L_2–L_∞ state estimation problem for a class of delayed neural networks. Attention is focused on the design of a full-order state estimator such that the prescribed L_2–L_∞ performance constraint can be ensured. By utilizing the time-delay information sufficiently, a novel L_2–L_∞ performance analysis approach is proposed in this paper for the first time. Based on such an approach, the less conservative sufficient conditions are established in terms of linear matrix inequalities under which the L_2–L_∞ performance level can be achieved for the estimation error dynamics. Several numerical examples show that the proposed approach in this paper is explicitly effective in reducing the possible conservatism.

1. Introduction

Nowadays, neural networks have been widely applied in many fields, such as pattern recognition, associative memory, signal processing and solving optimization problem [22,25]. Considering that the transmission delays between the neurons are always inevitable and, it is thus more interesting to investigate the neural networks with time delays [14]. In many specific applications, the equilibrium points of neural networks are required to be stable. Moreover, the neuron states should be obtained from network measurements for certain applications. Therefore, two interesting problems have been well considered for delayed neural networks over the past two decades. One is the stability/passivity analysis problem [2,5,8,15,18,27,41,44,45] and the other is the state estimation/filtering problem [6,26,28,34,39,40,47]. On the other hand, it has been recognized that the linear matrix inequality (LMI) approach is more convenient for the analysis and design of delayed dynamical systems. In addition, when the size of time delay is small, it has been identified that delay-dependent results are generally less conservative than delay-independent ones.

As for the state estimation/filtering and control problems for dynamical systems with or without time delays, it is often the case that the system performance should be considered to deal with the exogenous disturbances [4,12,13,19–21,24,29,30,35,36,42,43,48]. The main objective of the celebrated Kalman filtering is to minimize the variance of the estimation error [21,24]. However, it is worth mentioning that the Kalman filtering is based on the known statistics of the Gaussian noises. For unknown but energy bounded disturbances, one can resort to the H_∞ and L_2–L_∞ (is called as l_2–l_∞ in a discrete-time framework) performance indexes. The aim of the H_∞ estimator/filtering is to ensure that the L_2 (l_2) gain from the exogenous disturbances to the estimation error is less than a prescribed positive scalar [12,19,20]. The L_2–L_∞ (l_2–l_∞) filtering is to minimize the peak value of the estimation error for all possible energy bounded disturbances, which is also referred as the energy-to-peak filtering or the generalized H_2 filtering [4,12,13,35,36,43]. Generally speaking, when the peak value of the filtering error is required to be as small as possible, the L_2–L_∞ (l_2–l_∞) filtering is the better choice [43].

Over the past years, the H_∞ and L_2–L_∞ (l_2–l_∞) state estimation/filtering problems have also received considerable attention for neural networks with or without time delays [1,10,11,16,17,23,37,38,46]. For example, the delay-dependent H_∞ and L_2–L_∞ filtering problems have been addressed in [16] for a class of neural networks with time-varying delay. In [10], the L_2–L_∞ filtering problem has been considered for Takagi–Sugeno fuzzy neural networks with constant delay by using Wirtinger-type integral inequalities and, in [11], the exponential dissipative and l_2–l_∞ filtering problems have been investigated for a class of discrete-time switched neural networks with constant delay.

* Corresponding author.
E-mail addresses: qwei@hpu.edu.cn, qweihpu@163.com (W. Qian).

http://dx.doi.org/10.1016/j.neucom.2017.08.027
0925-2312/

Up to now, in spite of the fruitful results made on L_2–L_∞ (l_2–l_∞) estimator/filter design for delayed dynamical systems (including delayed neural networks), it should be pointed out that the existing analysis approach remains conservative to some extent. Specifically, to ensure the L_2–L_∞ (l_2–l_∞) performance requirement, the proposed Lyapunov–Krasovskii (L–K) functional is bounded by the simple term $e^T(t)Pe(t)$ and then one delay-independent LMI condition is induced. It is obvious that the time-delay terms have been completed neglected when bounding the L–K functional and the resulting conditions are conservative especially for the case of small delay. Therefore, it is more challenging to develop the less conservative approach to address the L_2–L_∞ (l_2–l_∞) state estimation/filtering problem for delayed dynamical systems.

This paper revisits the L_2–L_∞ state estimation problem for a class of neural networks with time-varying transmission delay. Compared with the existing L_2–L_∞ performance analysis approach, the time-delay terms are reserved in this paper when bounding the L–K functional and all induced conditions are delay-dependent to ensure the L_2–L_∞ performance constraint. Finally, several numerical examples demonstrate the effectiveness of the novel analysis approach in reducing the conservatism. The main contributions of this paper are given as follows: (1) by utilizing the time-delay information sufficiently, a novel L_2–L_∞ performance analysis approach is proposed for the first time; and (2) under the proposed approach, the less conservative L_2–L_∞ design conditions of state estimator are established for delayed neural networks.

Notation. The superscript "T" denotes the transpose of a matrix. $\mathbb{R}^n$ represents the n-dimensional Euclidean space. $L_2[0, \infty)$ is the space of square integrable vector functions over an interval $[0, \infty)$. $P > 0$ means that P is a real symmetric and positive definite matrix. I denotes an identity matrix with proper dimension. The symmetric terms in a symmetric matrix are denoted by $*$.

2. Problem formulation

Consider the following neural network with n neurons and time-varying transmission delay:

$$\begin{cases} \dot{x}(t) = -Ax(t) + W_0 g(x(t)) + W_1 g(x(t-h(t))) \\ \qquad + J + B_1\omega(t), \\ y(t) = Cx(t) + Dx(t-h(t)) + B_2\omega(t), \\ z(t) = Hx(t), \end{cases} \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \mathbb{R}^n$ is the state vector of the neural network, $y(t) \in \mathbb{R}^m$ is the network measurement, $z(t) \in \mathbb{R}^p$ is the signal to be estimated, $\omega(t) \in \mathbb{R}^q$ denotes the noise disturbances belonging to $L_2[0, \infty)$, $g(x(t)) = [g_1(x_1(t)), g_2(x_2(t)), \ldots, g_n(x_n(t))]^T \in \mathbb{R}^n$ represents the neuron activation function with $g(0) = 0$, $A = \mathrm{diag}\{a_1, a_2, \ldots, a_n\}$ is a diagonal matrix with $a_i > 0$ $(i = 1, 2, \ldots, n)$, and W_0 and W_1 are the connection weight matrices, B_1, B_2, C, D and H are some given constant matrices with appropriate dimensions, and $J = [J_1, J_2, \ldots, J_n]^T \in \mathbb{R}^n$ is a constant input vector.

In addition, the function $h(t)$ denotes the time-varying transmission delay that satisfies

$$0 \le h(t) \le h, \quad \dot{h}(t) \le \mu, \tag{2}$$

where $h > 0$ and μ are some known scalars.

Throughout this paper, we assume that the neuron activation function $g(x)$ in delayed neural network (1) is bounded and satisfies the following Lipschitz condition:

$$|g_i(x) - g_i(y)| \le l_i|x - y|, \ \forall x, y \in \mathbb{R}, \ i = 1, 2, \ldots, n, \tag{3}$$

where l_i $(i = 1, 2, \ldots, n)$ are known positive scalars.

Under the assumption that the time delay $\tau(t)$ is available, we construct the following full-order state estimator:

$$\begin{cases} \dot{\hat{x}}(t) = -A\hat{x}(t) + W_0 g(\hat{x}(t)) + W_1 g(\hat{x}(t-h(t))) \\ \qquad + J + K[y(t) - \hat{y}(t)], \\ \hat{y}(t) = C\hat{x}(t) + D\hat{x}(t-h(t)), \\ \hat{z}(t) = H\hat{x}(t), \end{cases} \tag{4}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the estimator state, $\hat{y}(t) \in \mathbb{R}^m$ is the estimate of the measurement output, $\hat{z}(t) \in \mathbb{R}^p$ is the estimate of the signal $z(t)$ and K is the estimator gain matrix.

Denoting that $e(t) \triangleq x(t) - \hat{x}(t)$, $\tilde{z}(t) \triangleq z(t) - \hat{z}(t)$ and $\varphi(t) \triangleq g(x(t)) - g(\hat{x}(t))$, and using (1) and (4), we can obtain the following estimation error dynamics:

$$\begin{cases} \dot{e}(t) = -(A + KC)e(t) - KDe(t-h(t)) \\ \qquad + W_0\varphi(t) + W_1\varphi(t-h(t)) \\ \qquad + (B_1 - KB_2)\omega(t), \\ \tilde{z}(t) = He(t). \end{cases} \tag{5}$$

Moreover, it can be seen from (3) that the nonlinear vector function $\varphi(t) = [\varphi_1(t), \varphi_2(t), \ldots, \varphi_n(t)]^T$ in the error dynamics (5) satisfies the following condition:

$$|\varphi_i(t)| = |g_i(x_i(t)) - g_i(\hat{x}_i(t))| \le l_i|e_i(t)|, \\ \qquad i = 1, 2, \ldots, n. \tag{6}$$

Before presenting our main results, it is necessary to introduce the following important lemmas and definition.

Lemma 1. [33] *Let an $n \times n$ matrix $Z > 0$, two scalars a and b satisfying $b > a$ and a vector function $\rho(t) \in \mathbb{R}^n$ be given. If the integrations concerned are well defined, then the following two inequalities hold:*

$$(1) \quad (b-a)\int_a^b \rho^T(s)Z\rho(s)\,\mathrm{d}s \ge \left(\int_a^b \rho(s)\,\mathrm{d}s\right)^T Z \left(\int_a^b \rho(s)\,\mathrm{d}s\right),$$

$$(2) \quad \frac{(b^2-a^2)}{2}\int_{-b}^{-a}\int_{t+\theta}^{t} \rho^T(s)Z\rho(s)\,\mathrm{d}s\,\mathrm{d}\theta \ge \left(\int_{-b}^{-a}\int_{t+\theta}^{t} \rho(s)\,\mathrm{d}s\,\mathrm{d}\theta\right)^T Z \times \left(\int_{-b}^{-a}\int_{t+\theta}^{t} \rho(s)\,\mathrm{d}s\,\mathrm{d}\theta\right) \ (b > a \ge 0).$$

Lemma 2. [32] *For a given scalar $\alpha \in (0, 1)$, a given $n \times n$ matrix $Z > 0$ and two vectors $\zeta_1, \zeta_2 \in \mathbb{R}^n$, define the function $\Theta(\alpha, Z) = \frac{1}{\alpha}\zeta_1^T Z\zeta_1 + \frac{1}{1-\alpha}\zeta_2^T Z\zeta_2$. If there exists an $n \times n$ matrix M such that $\begin{bmatrix} Z & M \\ M^T & Z \end{bmatrix} > 0$, then the following inequality holds:*

$$\min_{\alpha \in (0,1)} \Theta(\alpha, Z) \ge \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix}^T \begin{bmatrix} Z & M \\ M^T & Z \end{bmatrix} \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix}.$$

Definition 1. For a given scalar $\gamma > 0$, the error dynamics (5) is said to be asymptotically stable with an L_2–L_∞ performance level γ if the error dynamics (5) with $\omega(t) = 0$ is asymptotically stable and, under zero-initial conditions ($e(t) = 0, t \in [-h, 0]$), the error dynamics satisfies $\|\tilde{z}(t)\|_\infty < \gamma\|\omega(t)\|_2$ for all nonzero $\omega(t) \in L_2[0, \infty)$, where

$$\|\tilde{z}(t)\|_\infty = \sup_t \sqrt{\tilde{z}^T(t)\tilde{z}(t)}, \quad \|\omega(t)\|_2 = \sqrt{\int_0^{+\infty} \omega^T(t)\omega(t)\mathrm{d}t}.$$