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MRI reconstruction via enhanced group sparsity and nonconvex regularization

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ABSTRACT

In this paper, a new approach to perform compressed sensing MRI (CS-MRI) reconstruction based on enhanced group sparsity and nonconvex regularization (GSNR) is presented. A new framework is developed at attempt to improve the group sparsity and the accuracy of estimated coefficients. To that end, first, we establish a generalized reordering model to train the optimal permutation, which reveals the inner structure of group and benefits to promote the sparsity of group for an arbitrarily fixed transform. Second, with the nonconvex log-sum regularization, a fast shrinkage operator to solve the corresponding nonconvex optimization problem is developed in which the optimal solution is accurately and quickly obtained. The effectiveness of GSNR is demonstrated for both noiseless and noisy real MR images. In both cases, the proposed algorithm generates high-quality images that are superior in terms of visual inspection and objective evaluations to the state-of-the-art approaches. In addition, the effects of reordering and nonconvex regularization are verified by simulations, respectively, to illustrate the superior performance of GSNR because of the proposed framework.

1. Introduction

Magnetic resonance imaging (MRI) provides a powerful and effective technique for current clinical diagnosis and scientific research due to its high resolution and noninvasiveness. One of the technical challenges faced by cine MRI is to reduce the acquisition time enabling the high spatio-temporal resolution imaging of a cardiac volume within a short scan time. Normally, undersampling k-space is a good means to accelerate imaging time for MRI. However, undersampling violates the Nyquist sampling rule, resulting in artifacts in reconstructed magnetic resonance (MR) images. To overcome this issue, a new promising approach termed as compressed sensing (CS) [1,2] has been extensively investigated and widely applied in signal processing, inverse problems and medical imaging. The theory of CS states that if a signal has sparse representation in a given transform domain, it is possible to reconstruct signals/images from fewer measurements with little or no information loss than that expected from the Nyquist sampling criterion. From the excellent performance produced by CS, it has been quickly applied to accelerate MRI research in some specific areas, for example, 2D MRI [3,4], dynamic MRI [5], and other MRI applications [6,7], by exploiting the sparse characteristic of image in a certain transform domain.

It is well-known that finding a sparse representation is vital to ensure the successful CS-MRI reconstruction, and studies show that higher sparsity of the image in the transform domain will lead to better quality of the reconstruction results [8,9]. In most cases, CS-MRI reconstruction methods utilize the sparsity of entire image in a pre-defined transform domain. For example, the finite difference as a sparse transform is studied, which is the famous total variation (TV) regularization [4,10], and discrete cosine transform (DCT), discrete wavelet transform (DWT) and contourlet transform to enforce the sparsity of images during the CS-MRI reconstruction, are also studied in [3,11,12]. From the results reported, these methods based on image-level sparsity constraints are applicable to characterize only a few features and some artifacts may appear in the reconstructed results due to the lack of adaptation in the predefined transforms. Several approaches are developed to suppress the artifacts by exploring the combination of the fixed transform domains, but the improvement is still limited [13,14].

To find a better sparse representation, K-SVD [15] is developed to train a patch-based dictionary, which utilizes the local sparsity of patch. It is also reported in [16] that patch-based sparse representation effectively characterizes local image features via dictionaries adaptively learnt from k-space data. Motivated by this, several methods are subsequently proposed to train adaptive

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dictionaries or transforms [17,18], which represent patches more sparsely and obtain more promising reconstructions. The issue of those methods is, however, that sparse coding is separately operated on each patch and the correlation among patches is not explored.

Recent developments also reveal that the nonlocal similarity among patches plays an important role in image recovery and by utilizing this features, the reconstructions show significant improvements [19-21]. One of the pioneer work exploring the similar patches for target patch is block-matching and 3D filtering (BM3D) [19] for image denoising. In the BM3D, a powerful 3D transform which is realized by implementing multiple DWTs or DCTs on each dimension of group is developed to sparsely represent the image. Thanks to the utilization of the similarity of the patches in the group, 3D transform achieves higher sparsity than patch-based 2D sparse representation [22]. The BM3D presents an important enlightenment on how to combine the local sparsity and nonlocal similarity to make full use of prior information. Inspired by the group sparsity, this idea has been also validated in the applications of CS-MRI reconstruction [22-24]. Compared with the patch-based CS reconstruction, group-based methods demonstrate great improvements in terms of the details preservation and artifacts removal. Similarly, low-rank regularization for group recovery also takes the advantages of the nonlocal similarity, and its intrinsic equivalence to the group sparsity has been proved [25]. In [26], the low-rank regularization has been applied to CS-MRI and achieves the state-of-the-art performance.

In the group-based sparse representation, the orthogonal transforms are generally utilized to implement sparse coding [22,23]. To enhance group-based sparsity, the idea of adaptive transforms is developed, and among them, the K-SVD is representative that trains all bases in a global dictionary, but involves large-scale optimization problem with high computational complexity. Different from training dictionary, a novel reordering technique is also proposed to overcome the shortage of general transforms [27]. This idea works as follows. First, a permutation is learnt from the pre-reconstructed signal. Second, based on this permutation, the signal is reordered to be more suitable for the pre-defined transform. The primary mechanism of this scheme is that the trained permutation carries structural information of image, and the pre-defined transform combined with reordering can thus adaptively represent the structure of image. Based on this concept, several approaches are proposed to reorder pixels in the image in terms of intensities to promote the quality of recovery results when TV or DCT is used as the fixed bases [27,28]. Furthermore, patch-based directional wavelet (PBDW) [29] is proposed to perform geometric transform on patches before wavelet sparse representation to improve the sparsity patch. Unfortunately, these methods only adopt conventional strategies to reorder pixels, and the improvement of sparsity is somewhat limited. Therefore, establishing a better framework to train the optimal permutation is urgent to better recover the image.

Once the sparsity is improved by the adaptive sparse transform, another important issue is how to regularize the sparsity of group in transform domain with the constraint of k-space data. The standard quantitative metric to promote sparsity is l_0 norm, which leads a NP hard problem. Nevertheless, it has been proved that the l_0 norm can be replaced with its closest convex surrogate, namely l_1 norm, to produce exact reconstructions, but with more measurements [2]. Because of the simplicity of the l_1 norm, numerous algorithms based on l_1-minimization have been proposed to successfully reconstruct MRI [3,12,13] via mature methods such as IST [30], ADMM [31]. In recent years, nonconvex functions are starting to attract people attentions in performing sparsity regularization. In [32,33], the nonconvex l_p $(0<p<1)$ norm and log-sum function are separately utilized as regularization terms, which yield more accurate results than that of l_1 norm. For the low-rank based model, nonconvex schatten p-norm in [34] and logdet in [26] are proposed to replace convex nuclear norm as the surrogate for the rank, respectively, and significant improvements are achieved. It is now safe to conclude that the nonconvex norm better approximates the l_0 norm and reduces the requirements for measurements [35,36].

In this paper, inspired by the nonconvex regularizations, we propose a new framework to take advantages of both reordering and nonconvex optimization. The main contributions of this work are as follows. First, we reanalyze the intrinsic relationship between reordering and sparsity, and based the analysis, a more general permutation matrix to define the reordering operation is proposed. Furthermore, the corresponding optimization problem to search the optimal permutation used to reorder patches of group is established. To increase the accuracy, the permutation calculation for each group is updated at each iteration. Second, the log-sum function as regularization term to accurately estimate transformed coefficients of group is adopted, and a new shrinkage function to efficiently solve the corresponding nonconvex optimization problem is designed. The analysis also shows that the proposed approach is computationally efficient. Finally, experimental results on in vivo MR images illustrate that the proposed algorithm based on group sparsity and nonconvex regularization (GSNR) provides the state-of-the-art performance in terms of both visual inspection and objective evaluation, compared with other approaches.

The remainder of this paper is organized as follows. The conventional CS-MRI reconstruction is briefly reviewed in Section 2. The new framework which combines group sparsity and nonconvex regularization is presented in Section 3. The proposed reordering scheme is provided in Section 3.2. The log-sum regularization and the corresponding shrinkage function are developed in Section 3.3. The experimental results are compared with current state-of-the-art methods and analyzed in Section 4. This paper concludes with a brief summary in Section 5. Proofs of mathematical formulas are presented in Appendix.

2. Review of traditional CS-MRI reconstruction

The general CS-MRI reconstruction model is studied, given by

$$\boldsymbol{y} = F_{\mathbf{u}}\boldsymbol{x} + \boldsymbol{v}, \tag{1}$$

where $\boldsymbol{y} \in \mathbb{C}^K$ is the undersampled k-space data, $\boldsymbol{x} \in \mathbb{C}^M$ is the discrete image to be reconstructed, $F_{\mathbf{u}} \in \mathbb{C}^{K\times M} (K \ll M)$ is the undersampled Fourier encoding matrix, and $\boldsymbol{v} \in \mathbb{C}^K$ is the additive noise vector. Utilizing the prior knowledge of the image $\boldsymbol{x}$, the solution $\hat{\boldsymbol{x}}$ of the above ill-posed inverse problem can be estimated from the undersampled k-space data $\boldsymbol{y}$ through solving the following minimization problem

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \frac{\lambda}{2}\|\boldsymbol{y} - F_{\mathbf{u}}\boldsymbol{x}\|_2^2 + \left\|\boldsymbol{\Phi}^H\boldsymbol{x}\right\|_p, \tag{2}$$

where $\|\boldsymbol{y} - F_{\mathbf{u}}\boldsymbol{x}\|_2^2$ is the data fidelity term, λ is the regularization parameter, $\|\boldsymbol{\Phi}^H\boldsymbol{x}\|_p$ is prior term which enforces the sparsity of the coefficient of the reconstructed $\boldsymbol{x}$ in a certain transform domain $\boldsymbol{\Phi}^H$ (for example, DCT, wavelet), and $\|\|_p$ stands for the l_p norm $(0 \le p \le 1)$. For patch-based sparse representation, the assumption is that arbitrary image patch $\boldsymbol{x}_i$ can be sparsely represented by $\boldsymbol{x}_i = \boldsymbol{\Phi}\boldsymbol{\alpha}_i$ [16], where $\boldsymbol{\alpha}_i$ denotes the sparse coefficients of $\boldsymbol{x}_i$. Therefore, the patch-based optimization becomes

$$\left(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\alpha}}\right) = \arg\min_{\boldsymbol{x},\boldsymbol{\alpha}} \frac{\lambda}{2}\|\boldsymbol{y} - F_{\mathbf{u}}\boldsymbol{x}\|_2^2 + \frac{\beta}{2}\sum_{i=1}^{N}\|\mathbf{R}_i\boldsymbol{x} - \boldsymbol{\Phi}\boldsymbol{\alpha}_i\|_2^2 + \sum_{i=1}^{N}\|\boldsymbol{\alpha}_i\|_p, \tag{3}$$

where $\mathbf{R}_i$ stands for a matrix operator extracting the patch at position i, i.e., $\boldsymbol{x}_i = \mathbf{R}_i\boldsymbol{x}$, β is a penalty parameter. To obtain the solution,