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A reduced-order approach to filtering for systems with linear equality constraints

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ABSTRACT

In this paper, the filtering problem is investigated for a class of discrete systems with linear equality constraints. The system under consideration is subject to both noises and time-varying constrained conditions. Attention is focused on the design of a new reduced-order filter under a mild assumption such that the estimation performance of the proposed filter outperforms those of the traditional filters. By using the reorganized constraint information, the original system is transformed to a reduced-order system. A new recursive state estimator is developed, which is proved to have higher estimation precision than several existing filters. Subsequently, further analysis shows that the constrained Kalman predictor is a special case of the proposed filter. Finally, a numerical example is employed to demonstrate the effectiveness of our approach.

1. Introduction

In the past few decades, the filtering problems have been extensively investigated and successfully applied in many branches of engineering [1,12,13,15–17,36,39,40]. In the real world, a lot of physical systems are subject to constraints, which can arise from different reasons such as the basic laws of physics, geometry consideration of a system, and the mathematical description of the state vector [2,7,21,31–34]. In general, constraints are usually formulated as a series of mathematical expressions, which contain valuable prior information about the system state and should be taken into account in the system analysis or state estimation. It is well known that system states affected by *linear equality* constraints are often encountered in engineering applications such as target tracking and navigation [37,38], manufacturing production [22], engine health estimation [30], estimation of structural model [3,20], multi-sensor information fusion [41] and vehicle motion [27]. Therefore, in the past decade, many effective filtering methods have been proposed to apply these equality constraints within the Kalman filter framework; see, e.g., [11,29,35,42,43] and the references therein.

Among a variety of existing methods for the systems with equality constraints, the pseudo-observation method has been implemented in [41] by treating constraint equations as additional measurement equations. Thus, the state estimation problem has been converted into a regular filtering problem with two types of measurements. However, this method may lead to unstable filtering results since the augmented measurement noises have singular covariance. The results in [41] have been extended in [11] to the case of the nonlinear systems. The projection method is another popular technique to deal with the constrained problems [29,35], which has been presented by projecting the state estimation obtained from Kalman filtering onto the subspace spanned by the constraints. Unfortunately, this method bears no relation to the true constrained optimum since the projection process is only a correlation that forces the unconstrained estimation onto the constraint surface. Moreover, the null space filtering method has been presented in [14], where the system state is assumed to be a degenerate random vector which contains deterministic part and stochastic part. However, the precision of the algorithm in [14] is not high since some useful information has been lost in the estimation process.

To obtain more accurate estimation, some assumptions have been made and the constrained models have been reformulated in [22,42,43], where the filters for the new systems have been designed. For example, in [22], the system noises and measurement noises have been assumed to satisfy some constraint equations and a constrained Kalman predictor has been constructed, which has better performance than the projection filter and the unconstrained filter. A similar scheme has also been adopted in [42,43], where a new projection system has been derived by using an oblique projector on the null space of the constraint matrix and the covariance of system noise is assumed to be singular. This algorithm is more efficient than the

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pseudo-observation method. Unfortunately, the application scopes of the two methods mentioned above are narrow since these assumptions are too strict to be satisfied. Therefore, the purpose of this paper is to further study the filtering method for the constrained systems and design a high precision filter with relaxed assumptions.

In this paper, our aim is to address the state estimation problem for the systems with linear equality constraints and propose a new constrained filtering method. By using the least squares estimator and Kalman filter, a reduced-order estimator is first designed, and the performances of different filters are then compared. A simulation example is provided to show the effectiveness of the proposed state estimate scheme. The main contributions of this paper can be outlined as follows: (1) A novel reduced-order is proposed in order to solve the state estimation problem of the constrained systems. (2) The developed algorithm is proved to have higher estimation precision than the projection method, the unconstrained method and the null space method. (3) The new filter is designed under a mild assumption, and the constrained Kalman predictor developed under stronger assumptions is shown to be a special case of our approach.

The remainder of this paper is organized as follows. In Section 2, a dynamic system model with linear equality constraints is described, which is then transformed into a new system without constraints. The recursive estimator for the constrained state is presented in Section 3. Section 4 compares the new method with other three constrained algorithms and gives a degenerate form of our approach. A numerical example is shown in Section 5 and a conclusion is provided in Section 6.

2. Problem formulation and preliminaries

2.1. Problem formulation

Consider a class of linear systems described by the following model:

$$x_k = A_k x_{k-1} + w_{k-1} \tag{1}$$

$$z_k = H_k x_k + v_k \tag{2}$$

where $x_k \in \Re^n$ is the state vector satisfying constraints

$$D_k x_k = d_k \tag{3}$$

and $z_k \in \Re^m$ $(m \le n)$ is the measurement output. w_k and v_k are independent Gaussian white noises with covariance $U_k > 0$ and $R_k > 0$, respectively. $A_k \in \Re^{n\times n}$, $H_k \in \Re^{m\times n}$, $D_k \in \Re^{s\times n}$ and $d_k \in \Re^s$ are system matrix, observation matrix, constraint matrix and constraint vector, respectively. In this paper, D_k is assumed to have full row rank s. This assumption can always be satisfied by a linear transformation and renaming the constrained matrix D_k and vector d_k.

2.2. System transformation

Since $Rank\{D_k\} = s$, there exists an invertible matrix $D_{1,k}$ composed of s columns in D_k. Without loss of generality, we assume that the first s columns of D_k form $D_{1,k}$, and the last $n-s$ columns are denoted as $D_{2,k}$. Then, we have $D_k = [D_{1,k}\ \ D_{2,k}]$. Correspondingly, the system state is rewritten as $x_k = \begin{bmatrix} x_{1,k} \\ x_{2,k} \end{bmatrix}$, and (3) becomes

$$d_k = [D_{1,k}\ \ D_{2,k}] \begin{bmatrix} x_{1,k} \\ x_{2,k} \end{bmatrix} = D_{1,k}x_{1,k} + D_{2,k}x_{2,k}. \tag{4}$$

Since $D_{1,k}$ is invertible, Eq. (4) yields

$$x_{1,k} = D_{1,k}^{-1}(d_k - D_{2,k}x_{2,k}) \tag{5}$$

and

$$x_k = \begin{bmatrix} x_{1,k} \\ x_{2,k} \end{bmatrix} = \begin{bmatrix} -D_{1,k}^{-1}D_{2,k}x_{2,k} \\ x_{2,k} \end{bmatrix} + \begin{bmatrix} D_{1,k}^{-1}d_k \\ 0 \end{bmatrix}. \tag{6}$$

Substituting (6) into (1) gives

$$\overline{D}_k x_{2,k} = \overline{A}_k x_{2,k-1} + \overline{d}_k + w_{k-1} \tag{7}$$

where

$$\overline{A}_k = A_k \overline{D}_{k-1}, \overline{D}_k = \begin{bmatrix} -D_{1,k}^{-1}D_{2,k} \\ I \end{bmatrix},$$

$$\overline{d}_k = A_k \overline{D}_{1,k-1} d_{k-1} - \overline{D}_{1,k} d_k, \overline{D}_{1,k} = \begin{bmatrix} -D_{1,k}^{-1} \\ 0 \end{bmatrix}.$$

Similarly, substituting (6) into (2) yields a new measurement equation

$$z_k = H_k \overline{D}_k x_{2,k} + v_k = \overline{H}_k x_{2,k} + v_k \tag{8}$$

where $\overline{H}_k = H_k \overline{D}_k$.

Eqs. (7) and (8) form a new reduced-order system model without constraint that is similar to singular systems [19].

3. A reduced-order recursive estimator

In this section, we intend to establish a unified framework to deal with the addressed filtering problem for the systems with linear equality constraints. The prediction of $x_{2,k}$ is first calculated by using the filtering results at time instant $k-1$. Then, measurement (8) is used to update the prediction. The prediction and estimation of x_k are computed through (6).

3.1. Prediction of system's state

At time instant k, suppose that the estimation $\hat{x}_{2,k-1|k-1}$ and error covariance $P_{2,k-1|k-1}$ are known. To give the prediction of the state, the following lemma is needed.

Lemma 1 (*Nobe and Daniel [26]*). *Consider matrices $\alpha, \beta, R \ge 0$ and vector x with appropriate dimensions. The optimization problem*

$$\min_x (\alpha x - \beta)^T R(\alpha x - \beta)$$

has a unique solution if and only if the matrix $\alpha^T R \alpha$ is non-singular, and the optimal solution is given by $\hat{x} = (\alpha^T R \alpha)^{-1} \alpha^T R \beta$.

For system (7), the prediction problem can be described as

$$\hat{x}_{2,k|k-1} = \arg\min_{x_{2,k}} \left(\overline{D}_k x_{2,k} - \overline{A}_k x_{2,k-1} - \overline{d}_k\right)^T U_{k-1}^{-1} \left(\overline{D}_k x_{2,k} - \overline{A}_k x_{2,k-1} - \overline{d}_k\right).$$

Furthermore, with $\hat{x}_{2,k-1|k-1}$ and $P_{2,k-1|k-1}$, the above problem becomes

$$\hat{x}_{2,k|k-1} = \arg\min_{x_{2,k}} (\overline{D}_k x_{2,k} - b_k)^T \overline{U}_{k-1}^{-1} (\overline{D}_k x_{2,k} - b_k) \tag{9}$$

where

$$b_k = \overline{A}_k \hat{x}_{2,k-1|k-1} + \overline{d}_k, \overline{U}_{k-1} = U_{k-1} + \overline{A}_k P_{2,k-1|k-1} \overline{A}_k^T.$$

Since $\overline{U}_{k-1}$ is positive definite and $\overline{D}_k$ is full column rank, it can be concluded that $\overline{D}_k^T \overline{U}_{k-1}^{-1} \overline{D}_k$ is invertible. With Lemma 1, the optimal solution for (9) is

$$\hat{x}_{2,k|k-1} = \left(\overline{D}_k^T \overline{U}_{k-1}^{-1} \overline{D}_k\right)^{-1} \overline{D}_k^T \overline{U}_{k-1}^{-1} \overline{A}_k \hat{x}_{2,k-1|k-1} \tag{10}$$

whose error covariance is

$$P_{2,k|k-1} = \left(\overline{D}_k^T \overline{U}_{k-1}^{-1} \overline{D}_k\right)^{-1}. \tag{11}$$