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Novel pinning control strategy for coupled neural networks with communication column graphs

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ABSTRACT

This paper deals with the problem of chaotic synchronization for coupled neural networks by a novel control strategy. Additional communication control graphs which prescribe the information flow available for controls purposes are introduced, and such pinning system has not been investigated before. Many relaxed information are used to construct augmented Lyapunov–Krasovskii functional (LKF), which makes use of more relax variables by employing the new type augmented matrices with Kronecker produce operation. By resorting to Lyapunov function methods and analysis techniques, the tasks to get the pinning synchronization of dynamical networks are solved in terms of a set of LMI inequalities, which are easy to be analyzed or tested. Finally, numerical simulations are performed to demonstrate the effectiveness of the analytical results.

1. Introduction

Neural networks have attracted the attention of many researchers from different areas since they have been successfully applied in many fields including signal and image processing, associative memories, combinatorial optimization, automatic control, and so on (see [1–8], references therein).

Recently, the research on synchronization and dynamical behavior analysis of coupled neural networks has become an important direction [9–11]. So it is necessary to develop a method to study the synchronization dynamical behavior of coupled neural networks, which can help to improve the stability, safety and efficiency of the system.

In many circumstances, coupled neural networks cannot synchronize by themselves; thus, some control strategies should be adopted to achieve synchronization. Pinning control, as an important control technique, in the past ten years, has been widely used to synchronize complex dynamical networks. On the one hand, Pinning control has been developed as an effective method to design controller feedback gains. For complex systems with coupling delays, various techniques have been utilized to deal with the pinning synchronization, such as [12–16]. However, in these papers, there still exist some fundamental and yet challenging problems in pinning control of coupled networks: (1) What kinds of pinning schemes should be chosen for a given complex network to achieve synchronization? (2) What types of controllers should be designed to ensure the synchronization? (3) How to solve the pinning synchronization problems for neural networks with hybrid coupling?

Motivated by the results of the mentioned discussions, in this paper, we will study the hybrid synchronization of coupled general complex dynamical networks with nonlinear and linear coupling via pinning control. We propose a novel pinning control strategy to solve synchronization problems for coupling neural networks. Furthermore, by employing augmented LKF, we can handle multitude Kronecker product terms. Combining the pinning control method with linear matrix inequality technique, some sufficient conditions for the hybrid synchronization of the drive and response networks.

Notations: R^n is the n-dimensional Euclidean space; $R^{m\times n}$ denotes the set of $m\times n$ real matrices. I_n represents the n-dimensional identity matrix. The notation $X\geq 0$ (respectively, $X>0$) means that X is positive semidefinite (respectively, positive definite). The notation $A\otimes B$ stands for the Kronecker product of matrices A and B; $diag(\cdots)$ denotes a block-diagonal matrix; $\begin{bmatrix} X & Y \\ * & Z \end{bmatrix}$

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stands for $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix}$. Matrix dimensions, if not explicitly stated, are assumed to be compatible for algebraic operations.

2. Preliminaries

In this section, we introduce a novel pinning control strategy for coupled neural networks. Consider the following control system:

$$\begin{aligned}\dot{x}_i(t) &= -Cx_i(t)+Af(x_i(t))+Bf(x_i(t-\tau(t)))+I(t)\\ &\quad+\sum_{j=1}^{N}G_{ij}^{(1)}D_1x_j(t)+\sum_{j=1}^{N}G_{ij}^{(2)}D_2f(x_j(t-\tau(t)))+u_i(t),\ i=1,2,\ldots,l,\\ \dot{x}_i(t) &= -Cx_i(t)+Af(x_i(t))+Bf(x_i(t-\tau(t)))+I(t)\\ &\quad+\sum_{j=1}^{N}G_{ij}^{(1)}D_1x_j(t)+\sum_{j=1}^{N}G_{ij}^{(2)}D_2f(x_j(t-\tau(t))),\ i=l+1,l+2,\ldots,N,\end{aligned} \tag{1}$$

where $x_i(t)=(x_{i1}(t),x_{i2}(t),\ldots,x_{in}(t))^T\in R^n$ is the neuron state vector of the ith network at time t. $f(x_i(t))=(f_1(x_{i1}(t)),f_2(x_{i2}(t)),\ldots,f_n(x_{in}(t)))^T$, $I(t)=(I_1^T(t),I_2^T(t),\ldots,I_n^T(t))^T\in R^n$, $C=diag(c_1,c_2,\ldots,c_n)>0$ is the state feedback coefficient matrix, $A,\ B\in R^{n\times n}$ represent the connection weight matrix, $G^{(q)}=(G_{ij}^{(q)})_{N\times N},(q=1,2)$ represent the coupling connections; $D_1,D_2\in R^{n\times n}$ represent the inner coupling matrix and the discrete-delay inner coupling matrix. $\tau(t)$ is time-varying delay, and satisfy

$$0\le\tau(t)\le\tau,\quad 0\le\dot{\tau}(t)\le\mu<1,$$

τ and μ are known constant scalars.

$u_i(t),i=1,2,\ldots,N$, are the controllers to be designed. We design the linear state feedback controllers by

$$\begin{aligned}u_i(t) &= \sum_{j=1,j\ne i}^{N}k_{ij}^{(1)}D_1(x_j(t)-x_i(t))\\ &\quad+\sum_{j=1,j\ne i}^{N}k_{ij}^{(2)}D_2f(x_j(t-\tau(t))-x_i(t-\tau(t))),\end{aligned} \tag{2}$$

where $k_{ij}^{(q)}>0,(q=1,2)$, for $i=1,2,\ldots,l$; and $k_{ij}^{(q)}=0$, for $i=l+1,l+2,\ldots,N$, $k_{ij}^{(q)}$ are the control gains.

System (2) can be rewritten as

$$u_i(t)=\sum_{j=1}^{N}L_{ij}^{(1)}D_1x_j(t)+\sum_{j=1}^{N}L_{ij}^{(2)}D_2f(x_j(t-\tau(t))). \tag{3}$$

Where matrix $L^{(q)}=(L_{ij}^{(q)})_{N\times N},(q=1,2)$ are defined as

$$\begin{cases}L_{ij}^{(q)}=k_{ij}^{(q)},i\ne j,\\ L_{ii}^{(q)}=-\sum\limits_{j=1,j\ne i}^{N}k_{ij}^{(q)},\quad i,j=1,2,\ldots,N.\end{cases}$$

The initial conditions are given by

$$x_i(s)=\Pi_{i0}(s)\in\Psi([-\tau,0],R^n),\quad i=1,2,\ldots,N.$$

For simplicity, let

$$\begin{aligned}&x(t)=(x_1^T(t),x_2^T(t),\ldots,x_N^T(t))^T,\\ &\mathbf{I}(t)=(I^T(t),I^T(t),\ldots,I^T(t))^T,\\ &F(x(t))=(f^T(x_1(t)),f^T(x_2(t)),\ldots,f^T(x_N(t)))^T,\\ &\tilde{U}(t)=(u_1^T(t),u_2^T(t),\ldots,u_N^T(t))^T.\end{aligned}$$

Combining with the sign $\otimes$ of Kronecker product, model (1) can be rewritten as

$$\begin{aligned}\dot{x}(t) &= -(I_N\otimes C)x(t)+(I_N\otimes A)F(x(t))+(I_N\otimes B)F(x(t-\tau(t)))+\mathbf{I}(t)\\ &\quad+(G^{(1)}\otimes D_1)x(t)+(G^{(2)}\otimes D_2)F(x(t-\tau(t)))+\tilde{U}(t)\end{aligned} \tag{4}$$

From Eq. (3), we can obtain

$$\tilde{U}(t)=(L^{(1)}\otimes D_1)x(t)+(L^{(2)}\otimes D_2)F(x(t-\tau(t))) \tag{5}$$

Remark 1. The coupling connection weights $G_{ij}^{(1)}$ and $G_{ij}^{(2)}$ represent two graphs which we call the physical coupling graphs. These graphs are fixed and the weights are prescribed. To design the controls $u_i(t)$ we introduce two additional graphs, namely the communication control graphs, with weights $k_{ij}^{(1)}$ and $k_{ij}^{(2)}$.

Throughout this paper, the following assumptions are needed.

Assumption 1. The outer-coupling configuration matrices of the complex networks satisfy

$$\begin{cases}G_{ij}^{(q)}=G_{ji}^{(q)}\ge 0,\quad i\ne j,q=1,2\\ G_{ii}^{(q)}=-\sum\limits_{j=1,j\ne i}^{N}G_{ij}^{(q)},\quad i,j=1,2,\ldots,N\end{cases}$$

Assumption 2 (*Liu et al. [17–19]*). For $j\in 1,2,\ldots,N$, $\forall s_1,s_2\in R$, $s_1\ne s_2$, the neural activation functions satisfy

$$\sigma_r^-\le\frac{f_j(s_1)-f_j(s_2)}{s_1-s_2}\le\sigma_r^+,$$

We define

$$\Delta_1=\mathrm{diag}(\sigma_1^+\sigma_1^-,\ldots,\sigma_n^+\sigma_n^-),\quad \Delta_2=\mathrm{diag}\left(\frac{\sigma_1^++\sigma_1^-}{2},\cdots,\frac{\sigma_n^++\sigma_n^-}{2}\right).$$

Remark 2. As discussed in [19], the constants σ_r^-,σ_r^+ in Assumption 2 are allowed to be positive, negative, or zero. Hence, the resulting activation functions could be nonmonotonic and more general than the usual sigmoid functions. In addition, when using the Lyapunov stability theory to analyze the stability, such a description is particularly suitable since it quantifies the lower and upper bounds of the activation functions that offer the possibility of reducing the induced conservatism.

Next, we give some useful definitions and lemmas.

Definition 1. System (4) is said to be pinning synchronized if the following holds:

$$\lim_{t\to\infty}\|x_i(t)-x_j(t)\|=0,\quad i,j=1,2,\ldots,N$$

Lemma 1. *Let $\otimes$ denote the notation of Kronecker product. Then, the following relationships hold:*

(1) $(\alpha A)\otimes B=A\otimes(\alpha B)$
(2) $(A+B)\otimes C=A\otimes C+B\otimes C$
(3) $(A\otimes B)(C\otimes D)=(AC)\otimes(BD)$
(4) $(A\otimes B)^T=A^T\otimes B^T$

Lemma 2 (*Liu et al. [20]*). *Let $e=(1,1,\ldots,1)^T$, $E_N=ee^T$, and $U=NI_N-E_N$, $M\in R^{n\times n}$, $x=(x_1^T,x_2^T,\cdots,x_N^T)^T$, and $y=(y_1^T,y_2^T,\ldots,y_N^T)^T$ with $x_k,y_k\in R^n,(k=1,2,\cdots,N)$, then*

$$x^T(U\otimes M)y=\sum_{1\le i<j\le N}(x_i-x_j)^TM(y_i-y_j)$$

Lemma 3. *(Jensen's inequality): For constant matrix $\Upsilon\in R^{n\times n},\Upsilon^T=\Upsilon>0$, scalar $\rho>0$ and vector function $\varpi:[0,\rho]\to R^n$, we have:*

$$\rho\int_0^\rho\varpi^T(s)\Upsilon\varpi(s)\,ds\ge\left(\int_0^\rho\varpi(s)\,ds\right)^T\Upsilon\left(\int_0^\rho\varpi(s)\,ds\right)$$

Lemma 4 (*Zhang et al. [21]*). *Let H be an $n\times n$ any real matrix, K is an $n\times n$ positive definite matrix, $e=(1,1,\ldots,1)^T$, $E_N=ee^T$, and $U=NI_N-E_N$, $x=(x_1^T,x_2^T,\cdots,x_N^T)^T$, and $y=(y_1^T,y_2^T,\ldots,y_N^T)^T$ with $x_k,y_k\in R^n,(k=1,2,\ldots,N)$ Then, for any vectors x and y with appropriate*