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Event-triggered consensus in nonlinear multi-agent systems with nonlinear dynamics and directed network topology

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ABSTRACT

The event-triggered sampling control scheme is motivated by applications of embedded microprocessors with limited computation and storage resources. This paper proposes a novel framework for consensus of multiagent systems with inherent nonlinear dynamics in general directed networks by a distributed event-triggered control mechanism. A kind of high-performance event which effectively avoids continuous communications among agents is designed to analytically determine when an agent should sample its current state. Two kinds of general algebraic connectivity are introduced to respectively investigate the global consensus in strongly connected networks and a class of networks containing directed spanning trees. Sufficient conditions are derived to reach a global consensus based on algebraic graph, matrix theory, and Lyapunov control approach. A numerical example is presented to demonstrate the correctness of the theoretical results.

1. Introduction

Cooperative collective behaviors in networks of autonomous agents have received considerable attentions in recent years. Consensus has grown interests in understanding the intriguing animal group behaviors, such as flocking and swarming, and has emerging broad applications in sensor networks, unmanned air vehicles formations and robotic teams. To coordinate with other agents in a network, an agent needs to share the information with their adjacent peers and agrees on a certain value of interests. Numerous contributions have been made in [1–7,16,17,23–27], and references therein. Most researches are mainly concerned with continuous feedback control strategies under the assumption of unlimited computation and memory resources equipped for each agent [4,16,17]. Nevertheless, an agent in many real networks has limited resources and it is expected to update its control input as few as possible. It is necessary to extend life span of agents by using discontinuous control scheme instead of continuous control ones.

In order to take advantages of high-speed computers, microelectronic and communication networks, it is preferable to use a digital controller, particular in aerospace systems and industries [14]. When the processors have limited resources, the event-triggered control approaches are more favorable [8–13]. The event-triggered control algorithms are updated at discrete time instants, which can be implemented on digital platforms. The transmission frequency and control update can be greatly reduced, which can save bandwidth while guaranteeing similar control performance as the continuous control ones. The event-triggered control method is aperiodic, which needs communications among agents when the predesigned individual event-triggered function triggers. These factors motivate researchers to design event-triggered control schemes for digital platforms. Wang and Lemnon examined the event-triggered broadcasting of information in the distributed networked systems and presented a decentralized approach for determining event-triggered thresholds for nonlinear subsystems [10,11]. Event-triggered control of multi-agent systems was studied by Dimarogonas et al. in [8], where the control updating depends on the ratio of a certain measurement error with respect to the norm of a function of the state. Fan et al., studied the distributed rendezvous problem with distributed controller and designed the basic event-triggered algorithm [15]. Zhu et al., considered the event-based consensus problem of general linear multi-agent systems, and an improved algorithm is proposed for determining event time sequences, which can reduce not only control updates, but also communication between neighboring agents in [28]. Li et al., investigate the event-triggered consensus in nonlinear networks, however, the Zeno-behavior could not be overcome when the consensus is reached [29].

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Event-based broadcasting for multi-agent average consensus is considered in [32]. Event-based leader-following consensus of multi-agent systems with input time delay is studied by Zhu and Jiang in [30]. The problem of event-triggered pinning control of complex networks with switching topologies is investigated in [31]. Zeno-free, distributed event-triggered communication and control for multi-agent average consensus is studied in [33]. The distributed event-triggered consensus for multi-agent systems with directed topologies is studied in [34]. Wen et al. studied the aperiodic sampled-data control for the sliding-mode control scheme of fuzzy systems with communication induced delays via the event-triggered method [39]. Lu et al. investigated the event-triggered synchronization in complex dynamical networks [35], self-triggered consensus in multi-agent systems with switching topologies [36], pinning synchronization of coupled dynamical systems with Markovian switching couplings [37] and convergence of analytic neural networks with event-triggered synaptic feedbacks [38]. The drawback of the existing works with regard to the event-triggered control approach can be summarized as: (i) the network topology is undirected [8,15,33,37] and (or) (ii) the agents' model is described as linear dynamics, neglecting agents' inherent nonlinear dynamics behaviors [8,10–12,15,28,30,32–34] and (or) (iii) the Zeno behavior can be excluded before the consensus is reached, while it could not be eliminated after the consensus is reached [8,15,28,29]. This will result in the performance degeneration of control systems and (or) (iv) Precise consensus could not be guaranteed, i.e., all agents can only converge to a neighborhood region of the consensus state [28].

Based on the above discussions, the main purpose of this paper is to propose a novel framework for the consensus of multiagent systems with nonlinear dynamics in a general directed network by an event-triggered control scheme. The contributions of this paper are: (i) The event-triggered control scheme is introduced to solve the consensus problem of multiagent systems. The nonlinear dynamics term is incorporated to describe the inherent dynamical behaviors of agents and the network topology is directed. (ii) A high-performance event which avoids continuous communication between neighborhood agents is designed to analytically determine when the agent should sample its data. (iii) The precise consensus can be achieved, and meanwhile the Zeno behavior can be excluded during the whole running process of the control systems. (iv) Some existing theoretical results can be seen as special cases of this paper.

2. Preliminaries

2.1. Algebraic graph theory

The communications among nodes is modeled as a weighted directed graph $\mathcal{G}=\{\mathcal{V},\mathcal{E},\mathcal{W}\}$, where $\mathcal{V}=\{1,2,\ldots,N\}$ is the set of nodes, $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ is the set of edges, and $\mathcal{W}=(w_{ij})\in R^{N\times N}$ is the weighted adjacency matrix of $\mathcal{G}$ with nonnegative adjacency elements w_{ij} and zero diagonal elements. If edge $(j,i)\in\mathcal{E}$, then node j is called a neighbor of node i, which means that node i can receive information from node j, and this edge is assigned a positive edge weight w_{ij}. The neighborhood index set of node i is denoted by $\mathcal{N}_i=j\in\mathcal{V}|(j,i)\in\mathcal{E}$, while we indicate with $\overline{N}_i=|\mathcal{N}_i|$ the number of neighbors of node i. The Laplacian matrix $L=(l_{ij})\in R^{N\times N}$ of the directed graph $\mathcal{G}$ associated with the adjacency matrix $\mathcal{W}$ is defined as: $l_{ij}=-w_{ij}\le 0, i\ne j$; $l_{ii}=\sum_{j=1,j\ne i}^{N}w_{ij}\ge 0$ [18].

2.2. Notations

Let Z, R, and C be the integer number set, the real number set, and the complex number set, respectively. For $\lambda\in C$, Re $(\lambda)\in R$ denotes its real part. I_m denotes the identity matrix of order m and O denotes the zero matrix with appropriate dimension. 1_N denotes the vector with all elements being 1. For a real symmetric matrix A, let $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ denote respectively its minimum and maximum eigenvalue, and write $A>0 (A<0)$ if A is a positive (negative) definite matrix. For a vector ξ, $\xi>0$ represents that all elements of ξ are positive. Denote $|M|$ the cardinality for the set M. Unless specifically mentioned, all referenced norms $\|\cdot\|$ used in this paper are 1-norm for vectors or matrices. $\|\cdot\|_2$ represents 2-norm (Euclid norm).

3. Problem formulations

Consider a general class of multiagent systems with N agents, labeled by $1,2,\ldots,N$. Each agent being equipped with a small embedded micro-processor and capability-limited onboard communication and actuation modules has limited energy resources and computing capability. The multiagent systems using a distributed event-triggered sampling strategy can be described by

$$\dot{x}_i(t)=f(x_i(t),t)+c\sum_{j\in\mathcal{N}_i}w_{ij}\Gamma(x_j(t^j_{k'})-x_i(t^i_k)),\tag{1}$$

where $t\in[t^i_k,t^i_{k+1})$, t^i_k and $t^j_{k'}$ respectively denote the latest sampling time instants of agent i and j before t. $x_i(t)=(x_{i1}(t),x_{i2}(t),\ldots,x_{in}(t))^{\mathrm{T}}\in R^{\mathrm{n}}$ is the state variable of the ith agent. $f(x_i(t),t)=(f_1(x_i(t),t),f_2(x_i(t),t),\ldots,f_{\mathrm{n}}(x_i(t),t))^{\mathrm{T}}:R^{\mathrm{n}}\times R\mapsto R^{\mathrm{n}}$ represents a continuous but not necessarily differentiable vector-valued function, which models the inherently self-nonlinear dynamics of the uncoupled ith agent. $c>0$ is the coupling strength among agents. The inner coupling matrix $\Gamma=\mathrm{diag}\{\gamma_1,\gamma_1,\ldots,\gamma_{\mathrm{n}}\}\in R^{\mathrm{n}\times\mathrm{n}}$ is a positive definite diagonal matrix, where the diagonal element $\gamma_j>0$ means that the agents can communicate through their jth component of their states.

Assumption 1 (*Yu et al. [20]*). The nonlinear function $f(x(t),t)$ satisfies $f(0,t)=0, \forall t$, and there exist three positive constants θ, ε and K such that for any vectors $x,y\in R^{\mathrm{n}}$, $\|f(x,t)\|\le K\|x(t)\|$ and

$$(x-y)^{\mathrm{T}}(f(x,t)-f(y,t))-\theta(x-y)^{\mathrm{T}}\Gamma(x-y)\le-\varepsilon(x-y)^{\mathrm{T}}(x-y).\tag{2}$$

Lemma 1 (*Yu et al. [19]*). *Suppose that the graph $\mathcal{G}$ is strongly connected, that is the Laplacian matrix L is irreducible. Then, $L1_N=0$ and there exists a vector $\xi=(\xi_1,\xi_2,\ldots,\xi_N)^{\mathrm{T}}>0$ such that $\xi^{\mathrm{T}}L=0$. In addition, there exists a matrix $\Xi=\mathrm{diag}\{\xi_1,\xi_2,\ldots,\xi_N\}>0$ such that $\hat{L}=(1/2)(\Xi L+L^{\mathrm{T}}\Xi)$ is symmetric, and $\sum_{j=1}^{N}\hat{L}_{ij}=\sum_{j=1}^{N}\hat{L}_{ji}=0$, $i=1,2,\ldots,N$.*

Definition 1 (*Yu et al. [19]*). For a strongly connected network $\mathcal{G}$ with Laplacian matrix L, the general algebraic connectivity is defined to be the real number

$$a_\xi(L)=\min_{x^{\mathrm{T}}\xi=0,x\ne 0}\frac{x^{\mathrm{T}}\hat{L}x}{x^{\mathrm{T}}\Xi x},\tag{3}$$

where $\hat{L}=(\Xi L+L^{\mathrm{T}}\Xi)/2, \Xi=\mathrm{diag}(\xi_1,\xi_2,\ldots,\xi_N)^{\mathrm{T}}, \xi=(\xi_1,\xi_2,\ldots,\xi_N)$ with $\xi_i>0$ for all $i\in\mathcal{V}$ and $\sum_{i=1}^{N}\xi_i=1$.

Lemma 2 (*Horn and Johoron [21]*). *For the matrices A, B, C, and D with appropriate dimensions, we have the following:*

1. $(\vartheta A)\otimes B=A\otimes(\vartheta B)$, *where ϑ is a constant*;
2. $(A+B)\otimes C=A\otimes C+B\otimes C$;
3. $(A\otimes B)(C\otimes D)=AC\otimes BD$;
4. $(A\otimes B)^{\mathrm{T}}=A^{\mathrm{T}}\otimes B^{\mathrm{T}}$.

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