Neurocomputing 185 (2016) 183–190

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Fast control optimization for switched linear systems based on harmony search algorithm

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ARTICLE INFO

Article history:
Received 11 January 2015
Received in revised form
13 October 2015
Accepted 17 December 2015
Communicated by Lixian Zhang
Available online 31 December 2015

Keywords:
Harmony search
Periodic switching
Equal-time-interval switching
Asymptotical stability
Convex combination

ABSTRACT

In this paper, we consider fast control optimization for switched linear systems based on Harmony Search algorithm. A necessary and sufficient condition for the asymptotical stability of switched linear systems via a special periodic switching, i.e., equal-time-interval switching, is derived. The conclusion is then extended to a more general class of periodic switched linear systems based on the convex combination conditions. The performance index of fast control for periodic switching law designing is employed, and Harmony Search algorithm is presented to address the related optimization problem under the premise of quadratic stability. Thus the fast control objective for periodic switched linear systems can be achieved. This makes the switching law designing become relatively simple. Finally, the simulation results demonstrate the effectiveness of the proposed approach.

1. Introduction

Switched systems have been studied extensively in theory and applications in recent years [1]. Switched systems represent a special class of hybrid systems which comprised a set of subsystems with different continuous dynamics, discrete dynamics, and logic decisions rule [2–7]. There is only one of the subsystems that is active at each time for the switched systems, thus 'when' to switch and 'what mode' to switch to is an important control issue for switched systems [8–10].

The designing of switching law is an effective method for the asymptotical stability of switched linear systems and it has been well explored in [2,11–30]. Note that a common quadratic Lyapunov function is employed for all subsystems so that the switched linear systems are asymptotically stable under arbitrary switching law in [18,19]. However, the arbitrary switching assumption is not suitable in general as a common quadratic Lyapunov function for all the subsystems may not exist or may be difficult to construct when it exists [2]. Hence, multiple Lyapunov functions or piecewise quadratic Lyapunov functions are employed for the study on the stability of switched linear systems with the assumption that each location has a Lyapunov function and should be continuous on the switching boundaries [22–26]. Unfortunately, there is no general method available.

Periodic switching law designing is a kind of time-dependent control strategy for switched systems, which may serve as an analytical model in practice and are effective when switching requirements are deterministic [27]. In contrast to the state-dependent switching law, the periodic switching law is easy to be realized in engineering and there is no need to test the system's trajectory constantly to determine whether the state of the systems satisfies the switching condition. We can see examples of periodic switched systems in power circuits with switching semiconductor devices, teams of autonomous agents that coordinate through periodic communication, networks of chaotic oscillators that synchronize through periodic coupling, etc. [27,28]. The periodic switching for switched linear systems has been extensively studied in [27–30]. Tokarzewski considered the stability of periodic switched linear system based on the Baker–Campbell–Hausdorff formula and on known theorems on localization on the complex plane of eigenvalues of a matrix which has a simple geometric interpretation and allows the provision of qualitative as well as quantitative stability conditions for the system in [27]. Roberson and Stilwell investigated the L_2 gain of periodic linear switched system in [28]. However, the time average-system must be asymptotically stable for the switched linear system considered in [28]. Xie and Wang [29] and Gao and Jing [30] considered periodical stabilization problems for switched linear systems. Unfortunately, the switching periodic and the parameters are

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http://dx.doi.org/10.1016/j.neucom.2015.12.057
0925-2312/

difficult to be determined for the proposed approach in [29,30]. To the best of our knowledge, how to select the switching period and the problem of fast control for the periodic switched linear systems have not been considered in most existing methods.

There has been a growing interest in recent years on Harmony Search (HS) algorithm, which is a meta-heuristic algorithm introduced by Geem et al. in 2001 [31]. HS algorithm mimics the musician's improvisation process to find a global optimal solution or a near optimal solution determined by an objective via searching experience and effective exploration. Due to its simplicity, generality and flexibility and its lower parameter sensitivity [32,33], HS algorithm has been successfully applied to various types of non-linear optimization engineering problems including reliability problems [34,35], energy system dispatch [36] and the design of the fuzzy controller [37–39]. Other applications of HS algorithm can be found in [40].

However, to the best of our knowledge, the switching law design using Harmony Search algorithm has not been concerned. To make up for it, this paper focuses on the problem of fast control optimization for periodic switched linear systems using Harmony Search algorithm. A special period switching, i.e., equal-time-interval switching is proposed for switched linear systems, and the conclusion is extended to a more general class of periodic switching linear systems based on the convex combination conditions. Under the premise of quadratic stability, the period switching law designing is optimized based on stochastic optimization approach. This gives an effective method for the selection of switching period and the parameters of the convex combination conditions for the control object.

The rest of this paper is organized as follows. In Section 2, a brief overview of the harmony search algorithm is given. In Section 3, a necessary and sufficient condition for the asymptotical stability of special periodic switching, i.e., equal-time-interval switched linear systems is derived, and then extended to the periodic switched linear systems in general. In Section 4, the key issues related to the optimization for the object based on the HS algorithm are discussed in detail. In Section 5, the simulation for the application examples is conducted to demonstrate the effectiveness of the proposed approach. Finally, a brief conclusion is presented in Section 6.

Notations: In this paper, $\lambda_i(A)$ denotes the ith eigenvalue of matrix A; $\mathrm{Re}(\lambda_i(A))$ denotes the real part of $\lambda_i(A)$; $\|\cdot\|_F$ and $\|\cdot\|_\infty$ denote the Frobenius norm and ∞ norm, respectively.

2. Harmony Search (HS) algorithm

HS algorithm is based on the musician's search for a better state of harmony judged by the corresponding value of the objective function evaluation. The optimization procedure of the HS algorithm consists of the following steps:

Step 1: Initialize the optimization problem and algorithm parameters.

The optimization problem is defined as

$$\begin{aligned} &\min \quad f(x) \\ &\text{s.t.} \quad x_{jL} \le x_j \le x_{jU} \quad (j = 1, 2, \ldots, N), \end{aligned} \tag{1}$$

where $f(x)$ is the object function; x_{jL} and x_{jU} are the lower and upper bounds for decision variables x_j, respectively; and N is the number of design variables.

Step 2: Initialize the harmony memory.

Harmony memory (HM) contains a set of harmony vectors (solutions) with sizes equal to HMS. Each comment of harmony vector x' is determined by the decision variables x_j $(j = 1, 2, \ldots, N)$, where x'_j is generated randomly by the following equation:

$$x'_j = x'_{jL} + (x'_{jU} - x'_{jL}) \times rand(0, 1), \tag{2}$$

where x'_{jL} and x'_{jU} are the lower and upper bounds for decision variables x'_j, respectively. Each harmony x' represents a solution vector, and the objective value is evaluated according to Eq. (1).

The HM matrix has the following form:

$$HM = \begin{bmatrix} x_1^1 & x_2^1 & \cdots & x_N^1 & | f(x^1) \\ x_1^2 & x_2^2 & \cdots & x_N^2 & | f(x^2) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_1^{HMS} & x_2^{HMS} & \cdots & x_N^{HMS} & | f(x^{HMS}) \end{bmatrix} \tag{3}$$

where $f(x^1), f(x^2), \ldots, f(x^{HMS})$ are the value of the objective function.

Step 3: Improvise a new harmony from the HM.

A new harmony vector x^{new} is generated from the HM, which is called improvisation. The jth component of the new harmony vector, i.e., x_j^{new} is determined by three rules: memory consideration, pitch adjustment and random selection. The procedure works as follows:

```
for each j ∈ [1, N] do
  if rand(0, 1) ≤ HMCR then
    x_j^new = x_j^i (i = 1, 2, ..., HMS) %memory consideration
    if rand(0, 1) ≤ PAR then
      x_j^new = x_j^new ± rand(0, 1) × bw %pitch adjustment
    end if
  else
    x_j^new = x_jL + rand(0, 1) × (x_jU − x_jL) %random selection
  end if
end for
```

where x_j^i is the jth component of x^i, bw is an arbitrary distance bandwidth.

Step 4: Update harmony memory.

If the fitness of the improvised harmony vector x^{new} is better than that of the worst harmony x^{worst}, replace the worst harmony in the HM with x^{new}.

Step 5: Check the stopping criterion.

If the maximal iteration number (NI) is satisfied, computation is terminated. Otherwise, Steps 3 and 4 are repeated.

3. Study on periodic switched linear systems

In this section, we first introduce the model of switched linear systems. Then, a necessary and sufficient condition for the asymptotical stability of special periodic switching, i.e., equal-time-interval switched linear systems is derived and extended to the periodic switched linear systems in general based on the convex combinations, which is quadratic stable for the periodic switched systems.

3.1. Problem description

Consider a switched linear system described by the differential equations

$$\dot{x}(t) = A_{\sigma(t)} x(t), \quad x(0) = x_0 \tag{4}$$

where $A_{\sigma(t)} \in R^{n \times n}$, $x \in R^n$ is the state vector with the initial state $x(0) = x_0$, $\sigma(t) : R^+ \leftarrow M := \{1, 2, \ldots, m\}$ is the switching law which is right piecewise continuous, and $M := \{1, 2, \ldots, m\}$ is the finite index set and $m > 0$ is the number of subsystems.

The switching sequence can be defined as

$$\sum = \{x_0; (t_0, A_{\sigma_{(t_0)}}), (t_1, A_{\sigma_{(t_1)}}), (t_2, A_{\sigma_{(t_2)}}), \ldots, t_k, A_{\sigma_{(t_k)}}), \ldots, | \sigma(t_k) \in M, k \in N\} \tag{5}$$