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Automatic determination of cutoff frequency for filter design using neuro-fuzzy systems

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ABSTRACT

The cutoff frequency of filter has a great influence on the image quality of positron emission tomography (PET). Understanding this physical phenomenon and developing some intelligent strategies to effectively determine cutoff frequency have both theoretical and practical significance. This paper proposes a new algorithm for automatically choosing filter cutoff frequency using neuro-fuzzy system. In the proposed method, wavelet theory is used to extract noise information which enhances the accuracy of cutoff frequency calculation and improves filtering performance. A neuro-fuzzy system is developed for modeling cutoff frequency function and adjusting weight values using gradient descent scheme. As a general method, the proposed approach is tested by using typical window functions. Results show that proposed techniques are effective and efficient for automatically determining cutoff frequency.

1. Introduction

Filtering in medical devices is widely used to improve the quality of reconstructed images. Statistical noise, due to an insufficient number of events, generally requires smoothing with a lowpass filter. On the other hand, the compensation for camera blur requires the use of an inverse filter. Unfortunately, these two goals are in opposition: the lowpass filter attenuates nonzero frequencies, whereas the inverse filter enhances them.

Restoration filters attempt to strike a compromise, combining smoothing and compensation into one filter. They use a parameter known as cutoff frequency, ω_c, to trade off the camera blurring correction with noise suppression. The optimal value of ω_c is characteristic for a given image, depending on the number of counts, camera and acquisition parameters, and the activity distribution.

The standard algorithm used for clinical image reconstruction in PET is filtered back-projection (FBP). The data acquired in PET studies usually suffer from the presence of significant statistical noise. This situation is further exacerbated by the effect of high frequency amplification of the ramp reconstruction filter in FBP. To reduce the noise in the reconstructed image, a low-pass window function is usually applied to the filter. The commonly used filters include Hamming filter, cosine filter, Butterworth filter and ramp filter [1].

Many filter functions have been suggested for FBP, most of which are windowing functions generally used in signal processing while the others are specifically designed for PET. In general, a single parameter, cutoff frequency ω_c, is used to modify the frequency response for the chosen filter function. The quality of a reconstructed image, in terms of noise, resolution, contrast and other measures, can vary greatly with the use of different filters and cutoff frequencies. A filter with a high cutoff frequency may attain high resolution and contrast, but is likely to keep noise to degrade the reconstructed image quality. Conversely, a filter with low cutoff frequency can effectively suppress image noise, but may overly smoothen the image, decrease contrast and eventually introduce ringing artifacts [2].

Currently, the filter functions for clinical medical reconstruction are usually chosen in an empirical method, based on a relatively small sample of images and filters. Clearly, this empirical method is not optimal, and furthermore, does not offer systematic method for choosing filter for varied applications. Previous studies with the goal of selecting an optimal filter function for FBP in PET have used lesion detectability as the metric for optimization [3–6]. However, evaluating performance for lesion detection necessitates a time-consuming observer study, using ROC methodology [3] or alternatives such as a computer observer [4]. Furthermore, even after the extensive effort on an observer study is made, there is no guarantee that an optimal filter cutoff frequency can be attained. Automated methods have been proposed which apply statistical tests to the object and noise power spectra in the projections, in

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order to ascertain the optimal cutoff frequency for a restoration filter [7].

This paper aims to develop a new automatic method to determine cutoff frequency in PET. Based on our previous work in [8], a neuro-fuzzy network system is employed to model the cutoff frequency function. On the one hand, fuzzy systems are fundamentally well fitted to model the uncertainty where both noise reduction and detail preservation are required. On the other hand, neural networks have the ability to learn from examples automatically. As a general method, the proposed neuro-fuzzy systems combining neural networks and fuzzy set theories can be employed as a powerful tool for the removal of Gaussian noise from PET images. To demonstrate the advantages and effectiveness of the proposed method, the experimental investigations are conducted and results are reported in this paper.

2. Background

2.1. Two-dimensional central-section theorem

The central-section theorem, also known as the central-slice or Fourier-slice theorem, is a foundational relationship in analytical image reconstruction. This theorem states that the Fourier transform of a one-dimensional projection is equivalent to a section, or profile, at the same angle through the center of the two-dimensional Fourier transform of the object. Fig. 1 shows a pictorial description of the central-section theorem where $\mathcal{F}_1(p(s,\theta))$ is the one-dimensional Fourier transform of a projection, $\mathcal{F}_2(f(x,y))$ is the two-dimensional Fourier transform of the image. The central-section theorem indicates that if we know $P(\omega,\theta)$ at all angles $0 \le \theta < \pi$, then we can fill in values for $F(u,v)$. The inverse two dimensional Fourier transform of $F(u,v)$ gives rise to $f(x,y)$.

2.2. Principle of filtered backprojection

The image reconstruction is to obtain a representation of an object in the direct space from its representation in a projection space. An essential step in image reconstruction is backprojection, which is the adjoint to forward projection process that forms the projections of the object. Fig. 1 shows the backprojection along a fixed angle, θ. Conceptually, backprojection can be described as placing a value of $p(s,\theta)$ back into an image array along the appropriate LOR. However, since the knowledge of where the values came from was lost in the projection step, the best we can do is to place a constant value into all elements along the LOR.

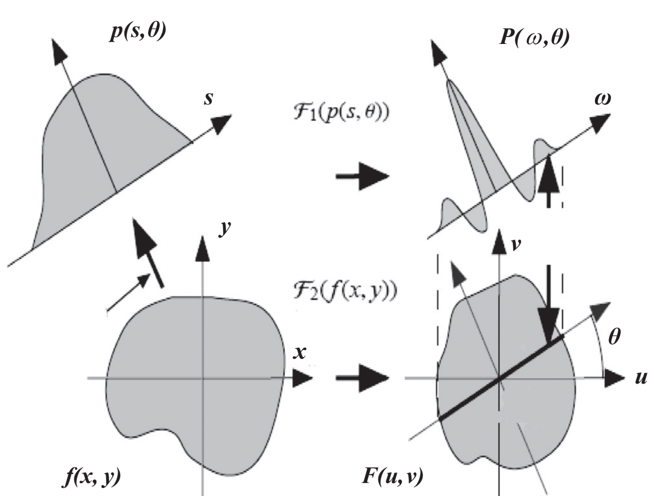

Fig. 1. Schematic of central slice theorem and spatial projection.

To make it simple, let us consider the case of parallel projections which can be constructed from the detection of annihilation pairs in PET. In two dimensions (2D), the whole set of parallel projections that can be built for every projection angle around the object is a 2D representation of the object in a projection space which is called sinogram. Conventionally, each line of sinogram corresponds to a parallel projection of the slice at a different angle θ. There is a close relationship between the representation of an object in the sinogram space and its representation in a spatial frequency space which permits by inversion of the Fourier Transform to reconstruct a representation of an object in direct space $f(x,y)$ from its projections $p(s,\theta)$. This relationship is expressed by the central slice theorem which connects the 1D Fourier Transform of a parallel projection $P(\omega,\theta)$ to the 2D Fourier Transform of the object image $F(u,v)$ along an axis perpendicular to the projection direction:

$$P(\omega,\theta) = F(\omega\cos(\theta), \omega\sin(\theta)). \tag{1}$$

Consequently, as shown in Fig. 1, measuring the projections around the object is equivalent to measuring the 2D Fourier Transform of the object using a polar coordinate system. Thus, the representation of the object in direct space can be obtained by inverting this frequency space representation. It is noted that the inverse Fourier Transform has to be applied in a Cartesian coordinate system. Therefore, a Jacobean has to be used to deal with the change of variables in the frequency space from polar coordinates to Cartesian coordinates. This is accomplished by multiplying the frequency space representation of the object obtained from the 1D Fourier Transform of the measured projections by the absolute value of the frequency $|\omega|$. In other words, a ramp filter is applied to the measured projections, and the representation of the object in the direct space is obtained after having backprojection of these filtered projections onto the lines of projections

$$f(x,y) = \int_0^{\pi} d\theta \int_{-\infty}^{+\infty} d\omega |\omega| e^{i2\pi\omega s}, \tag{2}$$

$$s = x\cos(\theta) + y\sin(\theta). \tag{3}$$

This filtered backprojection algorithm is a unique analytical solution to the problem of the inversion of the 2D Radon Transform and allows for reconstructing the image of a 2D object from its projections. This solution is purely analytical in the sense that projections are assumed to be continuous functions measured with an infinite accuracy.

It should be noted that the simple filter $|\omega|$ is an ideal filter. In practice, reconstructions usually are extremely noisy, as such other window options, such as the Hamming window, for reducing high frequencies, are preferable.

2.3. Window function and cutoff frequency

The ramp filter $|\omega|$ used in FBP is not related to sampling considerations. In order to reduce the amplification of high frequencies when ramp filter is used (statistical noise lies in the high frequencies), another low pass filter has to be added so as to take care of sampling problem. It is noted that projections are not continuously measured but sampled with a finite sampling step given by the scanner.

The filter is defined as the product of a low-pass smoothing function and is set to zero for frequencies larger than the Nyquist frequency. The two smoothing functions that we test are the Cosine and Hamming functions. The Cosine window function is defined as a function of frequency ω using

$$W(\omega) = \cos\left(\frac{\pi\omega}{\omega_c}\right) \quad \text{if } |\omega| \le \omega_c \tag{4}$$

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