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An analysis of global robust stability of delayed dynamical neural networks

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ABSTRACT

This paper studies the problem of establishing robust asymptotic stability of neural networks with multiple time delays and in the presence of the parameter uncertainties of the network. A new sufficient condition ensuring robust asymptotic stability is presented by manipulating the properties of some certain classes of real matrices and employing Homomorphic mapping and Lyapunov stability theorems. A numerical example is given to show that the condition obtained can outperform alternative ones in terms of conservatism and computational complexity.

1. Introduction

Dynamical neural networks have received remarkable consideration due to their important applications in various areas such as control engineering and optimization, signal processing, parallel computation, associative memories, and pattern recognition. In these applications, it is of crucial importance to know the dynamical behaviors of the designed neural networks. Therefore, stability analysis of neural networks has been an important topic in the past few decades. When studying the stability properties of neural networks, we should consider some key parameters that affect the dynamics of neural network systems. It is known that, in electronic implementation of neural networks, time delays are unavoidable due to the finite switching speed of neuron amplifiers, and the finite speed of signal propagation. Such time delays may change the dynamics of neural networks from stability to instability. On the other hand, in modelling of electronic neural networks, the numerical values of the network parameters may exhibit some unavoidable deviations due to the existence of modelling errors, external disturbance, and parameter fluctuations. These uncertainties in the network parameters may lead to some complex dynamical phenomena in neural networks. Therefore, it is indispensable to carry out the stability analysis of neural networks in the presence of time delays and parameter uncertainties. Such an analysis requires an investigation into the robust stability of neural networks.

Recently, many researchers have studied robust stability problem of neural networks and presented various sufficient criteria establishing existence, uniqueness and robust stability of equilibrium point for the class of delayed neural networks. Some previous literature results have focused on robust stability of neural networks of discrete, distributed or constant time delays. In the case of such time delays, it is useful to employ the linear matrix inequalities (LMIs) approach to derive the stability conditions [1–26]. However, when a neural network has multiple time delays, the linear matrix inequalities (LMIs) approach is not an appropriate tool for the analysis of stability. In the previous literature results on the stability of neural networks with multiple time delays basically rely on the M-matrix condition of interconnection matrices or establish some relationships between the network parameters [1–26].

In this paper, we study the robust stability of neural networks with multiple time delays. By manipulating the properties of some certain classes of real matrices, and constructing a novel Lyapunov functional together with using Homomorphic mapping theorem, a new robust stability criterion is derived for the addressed neural network model. This condition establishes a new relationship between the network parameters of the neural system and proved to be an alternative result to the previously published corresponding robust stability results. A numerical example will also be given to show the significance of the proposed criterion.

The rest of this paper is organized as follows: in Section 2, some preliminaries are given. Section 3 presents the condition for the existence, uniqueness, and global robust stability of the equilibrium point for system (1). In Section 4, a comparative numerical example is

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given to illustrate the effectiveness of the proposed result and a comparison is made between our result and the previous literature results. The concluding remarks are given in Section 5.

2. Preliminaries

In this section, we will first give some notations: throughout this paper, the superscript T represents the transpose. I will denote the unity matrix of appropriate dimension. For $x=(x_1,x_2,\ldots,x_n)^T$, $|x|$ will denote $|x|=(|x_1|,|x_2|,\ldots,|x_n|)^T$. For $A=(a_{ij})_{n\times n}$, $|A|$ will denote $|A|=(|a_{ij}|)_{n\times n}$, and $\lambda_m(A)$ and $\lambda_M(A)$ will denote the minimum and maximum eigenvalues of A, respectively. $A>0$ will imply that A is a symmetric matrix and positive definite. For $x=(x_1,x_2,\ldots,x_n)^T$ and $A=(a_{ij})_{n\times n}$, the following vector and matrix norms will be used in the proofs of the main result of this paper:

$$\|x\|_1=\sum_{i=1}^{n}|x_i|,\quad \|x\|_2=\left\{\sum_{i=1}^{n}|x_i|^2\right\}^{1/2},\quad \|x\|_\infty=\max_{1\le i\le n}|x_i|$$

$$\|A\|_1=\max_{1\le i\le n}\sum_{j=1}^{n}|a_{ji}|,\quad \|A\|_2=\left[\lambda_{\max}(A^TA)\right]^{1/2},$$

$$\|A\|_\infty=\max_{1\le i\le n}\sum_{j=1}^{n}|a_{ij}|$$

We can now proceed further to describe the neural system we consider. Dynamical behavior of neural networks with multiple time delays is governed by the following set of differential equations:

$$\frac{dx_i(t)}{dt}=-c_ix_i(t)+\sum_{j=1}^{n}a_{ij}f_j(x_j(t))+\sum_{j=1}^{n}b_{ij}f_j(x_j(t-\tau_{ij}))+u_i,\quad i=1,2,\ldots,n \tag{1}$$

where n is the number of neurons, $x_i(t)$ denotes the state of the neuron i at time t, $f_i(\cdot)$ denote activation functions, a_{ij} and b_{ij} denote the strengths of connectivity between neurons j and i at time t and $t-\tau_{ij}$, respectively; τ_{ij} represents the time delay required in transmitting a signal from the neuron j to the neuron i, u_i is the constant input to the neuron i, c_i is the charging rate for the neuron i.

A key issue in the stability analysis is the class of the activation functions employed in the design of neural networks as the nonlinear characteristics of activation functions has an important impact on determining stability conditions. In our stability analysis, we will consider the Lipschitz continuous nonlinear activation functions f_i defined by the following property:

$$|f_i(x)-f_i(y)|\le \ell_i|x-y|,\quad i=1,2,\ldots,n,\ \forall x,y\in R, x\ne y$$

where $\ell_i>0$ are positive Lipschitz constants. This class of functions will be denoted by $f\in\mathcal{L}$.

Another important factor that affects stability properties is the uncertainties of network parameters of the neural system. It is usually assumed that the deviations in the network parameters are within the certain intervals. In other words, the matrices $A=(a_{ij})_{n\times n}$, $B=(b_{ij})_{n\times n}$ and $C=diag(c_i>0)$ of system (1) are assumed to be norm-bounded within the following ranges:

$$\begin{aligned}
C_I&=[\underline{C},\overline{C}]=\{C=diag(c_i):0<\underline{c}_i\le c_i\le\overline{c}_i, i=1,2,\ldots,n\}\\
A_I&=[\underline{A},\overline{A}]=\{A=(a_{ij})_{n\times n}:\underline{a}_{ij}\le a_{ij}\le\overline{a}_{ij}, i,j=1,2,\ldots,n\}\\
B_I&=[\underline{B},\overline{B}]=\{B=(a_{ij})_{n\times n}:\underline{b}_{ij}\le b_{ij}\le\overline{b}_{ij}, i,j=1,2,\ldots,n\}
\end{aligned}\tag{2}$$

We will exploit the result of the following lemma in the derivation of the robust stability criterion for neural system (1):

Lemma 1 (*Ozcan and Arik [1]*). *Let A be any real matrix defined by*

$$A\in A_I=[\underline{A},\overline{A}]=\{A=(a_{ij})_{n\times n}:\underline{a}_{ij}\le a_{ij}\le\overline{a}_{ij}, i,j=1,2,\ldots,n\}$$

Then, for any two real vectors $x=(x_1,x_2,\ldots,x_n)^T$ *and* $y=(y_1,y_2,\ldots,y_n)^T$, *the following inequality holds*:

$$2x^TAy\le\gamma\sum_{i=1}^{n}x_i^2+\frac{1}{\gamma}\sum_{i=1}^{n}\varepsilon_iy_i^2$$

where γ *is any positive constant, and*

$$\varepsilon_i=\sum_{k=1}^{n}\left(\hat{a}_{ki}\sum_{j=1}^{n}\hat{a}_{kj}\right),\quad i=1,2,\ldots,n$$

with $\hat{a}_{ij}=\max\{|\underline{a}_{ij}|,|\overline{a}_{ij}|\}, i,j=1,2,\ldots,n.$

The results of the following lemmas will also be important in the context of this paper when comparing our results with the previously published corresponding literature results:

Lemma 2 (*Faydasicok and Arik [2]*). *Let the matrix A be intervalized as*

$$A\in A_I=[\underline{A},\overline{A}]=\{A=(a_{ij})_{n\times n}:\underline{a}_{ij}\le a_{ij}\le\overline{a}_{ij}, i,j=1,2,\ldots,n\}$$

Let $A^*=\frac{1}{2}(\overline{A}+\underline{A})$ *and* $A_*=\frac{1}{2}(\overline{A}-\underline{A})$. *Then*

$$\|A\|_2\le\sigma_1(A)=\sqrt{\||A^{*T}A^*|+2|A^{*T}|A_*+A_*^TA_*\|_2}$$

Lemma 3 (*Chen et al. [3]*). *Let the matrix A be intervalized as*

$$A\in A_I=[\underline{A},\overline{A}]=\{A=(a_{ij})_{n\times n}:\underline{a}_{ij}\le a_{ij}\le\overline{a}_{ij}, i,j=1,2,\ldots,n\}$$

Let $A^*=\frac{1}{2}(\overline{A}+\underline{A})$ *and* $A_*=\frac{1}{2}(\overline{A}-\underline{A})$. *Then*

$$\|A\|_2\le\sigma_2(A)=\|A^*\|_2+\|A_*\|_2$$

Lemma 4 (*Ensari and Arik [4]*). *Let the matrix A be intervalized as*

$$A\in A_I=[\underline{A},\overline{A}]=\{A=(a_{ij})_{n\times n}:\underline{a}_{ij}\le a_{ij}\le\overline{a}_{ij}, i,j=1,2,\ldots,n\}$$

Let $A^*=\frac{1}{2}(\overline{A}+\underline{A})$ *and* $A_*=\frac{1}{2}(\overline{A}-\underline{A})$. *Then*

$$\|A\|_2\le\sigma_3(A)=\sqrt{\|A^*\|_2^2+\|A_*\|_2^2+2\|A_*^T|A^*|\|_2}$$

Lemma 5 (*Singh [5]*). *Let the matrix A be intervalized as*

$$A\in A_I=[\underline{A},\overline{A}]=\{A=(a_{ij})_{n\times n}:\underline{a}_{ij}\le a_{ij}\le\overline{a}_{ij}, i,j=1,2,\ldots,n\}$$

Let $\hat{A}=(\hat{a}_{ij})_{n\times n}$ *with* $\hat{a}_{ij}=\max\{|\underline{a}_{ij}|,|\overline{a}_{ij}|\}$. *Then*

$$\|A\|_2\le\sigma_4(A)=\|\hat{A}\|_2$$

The result of the following lemma will be employed in the proof of the existence and uniqueness of the equilibrium point of neural network model (1):

Lemma 6 (*Ensari and Arik [4]*). *Consider the map* $H(x)\in C^0$. *If* $H(x)$ *satisfies the conditions*

(i) $H(x)\ne H(y)$ *for all* $x\ne y$,
(ii) $\|H(x)\|\to\infty$ *as* $\|x\|\to\infty$,

then, $H(x)$ *is homeomorphism of* R^n.

3. Robust stability analysis of equilibrium point

In this section, we will derive a new criterion establishing the global robust asymptotic stability of the equilibrium point of neural network defined by (1) under the parameter uncertainties given in (2).

Theorem 1. *Let the network parameters of the neural network model (1) satisfy (2) and* $f\in\mathcal{L}$. *Then, the neural system (1) is globally asymptotically robust stable, if there exist positive constants* α, β *and*

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