Neurocomputing ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Fuzzy Kalman-type filter for interval fractional-order systems with finite-step auto-correlated process noises

Shabnam Pourdehi *, Ali Azami, Faridoon Shabaninia

School of Electrical and Computer Engineering, Shiraz University, Iran

ARTICLE INFO

Article history:
Received 25 July 2014
Received in revised form
30 November 2014
Accepted 9 February 2015
Communicated by Cheng-Wu Chen

Keywords:
Kalman filter
Fractional-order system
Fuzzy
Interval
Auto-correlated noise

ABSTRACT

This paper considers the Kalman-type filter for a class of interval fractional-order systems with finite-step auto-correlated process noises. We propose a modified fuzzy Kalman-type filtering for such dynamic system. Fuzzy logic inference approach is used to handle interval characteristic of the system. Simulation example is given in order to demonstrate the effectiveness of the proposed algorithm.

1. Introduction

The Kalman filter has played an important role in systems theory and has found wide applications in many fields such as signal processing, control, and communications. A standard assumption in most of the existing state estimation algorithms is that the process noises are uncorrelated across time. However, this assumption is not always realistic. For example, in the target tracking system, the system state is usually consecutive, thus, when the process noises are dependent on the system state, the process noises are usually auto-correlated across time [1,2].

The fractional calculus (generalization of a traditional integer-order integral and differential calculus) idea has been mentioned in 1695 by Leibniz and L'Hospital [3]. In the end of 19th century Liouville and Riemann introduced first definition of fractional derivative. However, only just in late 60s of the 20th century this idea started to be interesting for engineers. Especially, when it was observed that the description of some systems is more accurate when the fractional derivative is used.

The effectiveness of nonlinear process control systems depends to a large extent on the quality of the model used for controller synthesis or tuning. Unfortunately, the choice of an adequate model for a nonlinear process and its parameterization involves difficulties in industrial practice. One of the more effective methods employed to describe real properties exhibited by many industrial processes, inclusive of those with distributed parameters, seems to be the description based on fractional-order derivatives. Many examples illustrating possible applications of such a description may be found in the literature. In so many researches, it is shown that fractional-order systems have better performances compared to classical ones, i.e. integer-order systems, especially where the nonlinear effects should be modeled [4–6]. The model allows introducing the nonlinear effects like friction and slipping in an easier way than any other dynamic model of integer-order. So, many real-world physical systems are better characterized by fractional-order differential equation [7–10]. For example, the electrochemical processes and flexible robot arm are modeled by fractional-order models. Even for modeling of traffic in information network fractional calculus is found to be a useful tool. More examples and areas of using fractional calculus are fractal modeling, Brownian motion, rheology, viscoelasticty, thermodynamics and others. It is shown that the fractional-order calculus plays an important role in analyzing thermodynamics, mechatronics systems, chemical mixing, and biological systems, electro-dynamics processes.

Another area of engineers interest, very fast developing, is the use of fractional-order controllers. The developed fractional-order model forms a basis for the model-based state feedback control. In order to apply the state-feedback control, when the state variables are not directly measured from the plant, an estimator is used to estimate the sate vector. The Kalman Filter is an optimal state vector estimator using the knowledge about the system model, input and output signals. Results of estimation are obtained by minimizing a minimum variance cost function in each step. Since the model of the plant is fractional-order. So, a new estimation

* Correspondence to: School of Electrical and Computer Engineering, Shiraz University, Post Box: (71348-51151- SHIRAZ- IRAN). Tel./fax: +98 71 32303081.
E-mail address: shabnamp_el83@yahoo.com (S. Pourdehi).

http://dx.doi.org/10.1016/j.neucom.2015.02.042
0925-2312/

tools appropriate for fractional-order models called Fractional Kalman filter (FKF) are needed [11–14]. In [11], as an extension to classical Kalman filter, fractional Kalman filter is derived for a fractional-order discrete-state space model with stochastic input.

Moreover, in many practical cases, the mathematical model is poorly known or uncertain [15,16]. In [15], Chen proposed incorporating bounded uncertainty in the system model directly as intervals, that is, describing the system matrices as matrices whose components are intervals. This type of uncertainty can easily arise in practice.

For example, when modeling from first principles, the values of certain physical parameters may not be known exactly, but known to lie within certain bounded limits with absolute certainty. Or when using system identification techniques to model a dynamic system, several models that differ only in the values of the matrix coefficients may be obtained under slightly different conditions, and these may all be contained in an interval. Therefore, in modeling and analysis of such systems, one needs to handle uncertainty. A few theoretical proposals have been published which proposed Kalman filter algorithm for interval linear system [15–19]. But, these proposed algorithms can be used only for integer-order system. In this paper, Kalman filter algorithm is designed for interval fractional-order system. In order to compensate the lack of precise knowledge of the system fuzzy logic system can be used. Fuzzy control has recently found extensive application for a wide variety of industrial systems and consumer products and has attracted the attention of many control researchers due to its model free approach. The fuzzy control algorithms attempt to make use of information from human experts. The expert information is generally represented by fuzzy terms, e.g., small, large, not very large, etc., for convenience or lack of more precise knowledge, ease of communication, and so forth. How ever, most of these fuzzy control algorithms are proposed without analytical tools for general design procedures to guarantee basic performance criteria. Generally, these fuzzy control approaches combine expert knowledge with the conventional engineering systems [20–29].

The goal of this paper is to improve the fractional Kalman filter presented in [11] and extend it to systems with finite-step auto-correlated process noises in the presence of uncertainties in the system matrices.

Motivated by the above discussion, an optimal robust Kalman-type recursive filter for a class of fractional systems with finite-step auto-correlated process noises subjected to uncertainties is proposed in this paper. Fuzzy logic inference approach is used to handle uncertainty or interval characteristic of the system. To the best of our knowledge, Kalman-type filter for a class of interval fractional-order systems with finite-step auto-correlated process noises had not been fully considered in the literature yet. Our paper is the first one that proposes Kalman filter for an interval fractional-order system. According to our literature review, Kalman filter had been designed for fractional-order systems with a strong and restrictive assumption that the process noise is an independent zero-mean white-noise sequence. In reality, however, the noise may not be white. In this paper we relax this assumption to the weaker assumption by designing Kalman filter for fractional-order system with finite-step auto-correlated process noises. Most of fractional-order systems do not satisfy the assumption of process with independent white-noise and the fractional Kalman filter proposed in [11–14] cannot be used in such cases. So, the proposed Kalman filter in our paper is implementable for a much larger class of fractional-order systems.

This paper outlined as follows. In Section 2, the problem statement, some necessary assumptions, and definitions are presented. Our main results and their rigorous proofs are described in Sections 3 and 4. In Section 5, to show the effectiveness of the proposed method, one numerical example is provided. Finally, concluding remarks are given in Section 6.

2. System description and problem formulation

The discrete linear fractional-order system with stochastic additive disturbances in state-space representation is given by the following set of equations:

$$x_k = Ax_{k-1} + Bw_{k-1} - \sum_{j=1}^{k} (-1)^j \gamma_j x_{k-j}$$
$$y_{k-1} = Cx_{k-1} + v_{k-1} \tag{1}$$

where

$$\gamma_j = diag\left[\binom{n_1}{j} \cdots \binom{n_N}{j}\right] \tag{2}$$

being $n_1, n_2, \ldots, n_N$ and N the orders and the number of system equations, respectively.

$x_k \in \Re^n$ is a state vector, $y_k \in \Re^p$ is a system output, and $A \in \Re^{n\times n}$, $B \in \Re^{n\times q}$ and $C \in \Re^{n\times p}$ are the state system input and system output matrices, respectively and $w_k \in \Re^q$ and $v_k \in \Re^p$ represent the system and measurement noise, respectively.

The noises w_k and v_k are uncorrelated with each other, and they together have the following statistical properties:

$$E[w_k] = E[v_k] = 0,$$
$$E\left[w_k w_l^T\right] = Q_k \delta_{k-l} + \sum_{i=1}^{f_k} Q_{k,l} \delta_{k-l+i} + \sum_{i=1}^{g_k} Q_{k,l} \delta_{k-l-i}, \tag{3}$$
$$E\left[v_k v_l^T\right] = R_k \delta_{k-l},$$
$$\delta_{i-j} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

It can be easily seen that the process noise is finite-step auto-correlated. For example, the process noise at time k is correlated with the process noises at time $k-g_k, \ldots, k-2, k-1$ and $k+f_k, \ldots, k+2, k+1$ with covariances $Q_{k,k-g_k}, \ldots, Q_{k,k-1}$ and $Q_{k,k+f_k}, \ldots, Q_{k,k+1}$ as well as respectively.

Let us start from redefinition of the linear fractional-order state space system. The redefinition introduces the new state vector, which collects g_k state vectors from time k to $k-g_k+1$.

The linear discrete fractional-order state space system in augmented form (the form with the new g_k-th elements state vector) is defined as follows:

$$X_k = \overline{A}X_{k-1} + \overline{B}w_{k-1} - \overline{I} \times \sum_{j=g_k+1}^{k} (-1)^j \gamma_j x_{k-j}$$
$$Y_{k-1} = \overline{C}X_{k-1} + v_{k-1} \tag{4}$$

where

$$X_{k-1} = \begin{bmatrix} x_{k-1} \\ x_{k-2} \\ x_{k-g_k} \end{bmatrix}, \quad \overline{I} = \begin{bmatrix} I \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$\overline{A} = \begin{bmatrix} (A+\gamma_1) & -(-1)^2\gamma_2 & \cdots & (-1)^{g_k}\gamma_{g_k} \\ I & 0 & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & \cdots & I & 0 \end{bmatrix},$$

$$\overline{B} = \begin{bmatrix} B \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \overline{C} = \begin{bmatrix} C & 0 & \cdots & 0 \end{bmatrix} \tag{5}$$

Please cite this article as: S. Pourdehi, et al., Fuzzy Kalman-type filter for interval fractional-order systems with finite-step auto-correlated process noises, Neurocomputing (2015), http://dx.doi.org/10.1016/j.neucom.2015.02.042