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Stability analysis of complex-valued neural networks with probabilistic time-varying delays

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A B S T R A C T

In this paper, the stability of complex-valued neural networks with probabilistic time-varying delays is investigated. Two important integral inequalities that include Jensen's inequality as a special case are developed. By constructing proper Lyapunov–Krasovskii functional and employing inequality technique, several delay-distribution-dependent sufficient conditions are obtained to guarantee the global asymptotic and exponential stability of the addressed neural networks. These conditions are expressed in terms of complex-valued LMIs, which can be checked numerically using the effective YALMIP toolbox in MATLAB. An example with simulations is given to show the effectiveness of the obtained results.

1. Introduction

Over the past few decades, neural networks have been widely studied due to their extensive applications in pattern recognition, associative memory, signal processing, image processing, smart antenna arrays, combinatorial optimization, and other areas [1]. In implementation of neural networks, however, time delays are unavoidably encountered [2]. It has been found that the existence of time delays often causes undesirable dynamic behaviors such as performance degradation, oscillation, or even instability of the systems [3]. Therefore, stability analysis of neural networks with time delays has received much attention, for example, see [3–7] and references therein.

As an extension of real-valued neural networks, complex-valued neural networks with complex-valued state, output, connection weight, and activation function become strongly desired because of their practical applications in physical systems dealing with electromagnetic, light, ultrasonic, and quantum waves [8]. In fact, complex-valued neural networks (CVNN) make it possible to solve some problems which cannot be solved with their real-valued counterparts. For example, the XOR problem and the detection of symmetry problem cannot be solved with a single real-valued neuron, but they can be solved with a single complex-valued neuron with the orthogonal decision boundaries, which reveals the potent computational power of complex-valued neurons [9].

In real-valued neural networks the activation functions are chosen to be smooth and bounded. In CVNN, if we choose an activation function to be smooth and bounded, then according to Liouville's theorem it will reduce to a constant. Therefore the choice of an activation function is the main challenge in CVNN. For different types of activation functions we need different approaches to study the relevant neural networks, which are quite different from those used for real-valued recurrent neural networks.

Recently, there have been some researches on the stability of various CVNN, for example, see [9–19] and references therein. In [9], authors proposed a CVNN and supposed that its weight matrix was Hermitian with nonnegative diagonal entries in order to preserve the stability of the network. And a computational energy function was introduced and evaluated in order to prove network stability for asynchronous dynamics. In [10], author weakened the assumption on weight matrix in [9], and derived a new stability condition. In [11], authors investigated the boundedness and complete stability of CVNN with constant delay, where the activation functions were chosen as $f(z)=\max(0,\mathrm{Re}(z))+i\max(0,\mathrm{Im}(z))$. In [12,13], a class of discrete-time recurrent neural networks were discussed, several sufficient conditions for stability of a unique equilibrium were obtained. In [14,15], the authors investigated the

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asymptotical stability and exponential stability for two types of CVNN with constant delay, where the activation functions either can be separated into their real and imaginary parts or satisfy the Lipschitz continuity condition in the complex domain. In [16,17], the asymptotical stability of CVNN with constant delay was investigated, where the activation functions can be expressed by separating their real and imaginary parts. In [18], authors considered a CVNN with time-varying delays and unbounded distributed delays whose activation functions can be expressed by separating their real and imaginary parts. In [19], when the activation functions satisfy the Lipschitz continuity condition in the complex domain, the asymptotical stability of CVNN with constant delay was studied.

It can be seen from the existing references that when investigating CVNN only the deterministic time-delay case was concerned, and the stability criteria were derived based only on the information of variation range of the time delay. As pointed out in [20], the time delay in some neural networks is often existent in a stochastic fashion, and its probabilistic characteristic, such as Poisson distribution or normal distribution, can often be obtained by statistical methods. It often occurs in real systems that some values of the delay are very large but the probabilities of the delay taking such large values are very small. In this case, if only the variation range of time delay is employed to derive the criteria, the obtained results may be somewhat more conservative [21–25]. Hence, research on neural network with probabilistic time-varying delays has both theory meaning and value of application. Recently, stability analysis of real-valued neural networks with probabilistic time-varying delays has been discussed, and some results related to this problem have been published, for example, see [26–29] and references therein. However, to the best of the author's knowledge, very few results on the problem of CVNN with probabilistic time-varying delays have been studied in the literature. This motivates our present research.

Motivated by the above discussions, the objective of this paper is to study the stability of CVNN with probabilistic time-varying delays. By employing a new Lyapunov–Krasovskii functional candidate and using matrix inequality technique, we obtain several sufficient conditions for checking the global asymptotic and exponential stability of CVNN with probabilistic time-varying delays.

Notations: The notations are quite standard. Throughout this paper, I represents the unitary matrix with appropriate dimensions; $\mathbb{C}$, $\mathbb{C}^n$ and $\mathbb{C}^{n\times m}$ denote, respectively, the set of all complex numbers, the set of all n-dimensional complex-valued vectors and the set of all $n\times m$ complex-valued matrices. A^* shows the complex conjugate transpose of complex-valued matrix A. The notation $X>Y$ means that X and Y are Hermitian matrices, and that $X-Y$ is positive definite. i shows the imaginary unit, i.e., $i=\sqrt{-1}$. For complex number $z=x+iy$, the notation $|z|=\sqrt{x^2+y^2}$ stands for the module of z. For complex-valued vector $z\in\mathbb{C}^n$, the notation $\|z\|$ is the Euclidean norm of z. Let $(\Omega,\mathcal{F},\{\mathcal{F}\}_{t\ge 0},\mathcal{P})$ be a complete probability space with a filtration $\{\mathcal{F}\}_{t\ge 0}$ satisfying the usual conditions (i.e., it is right continuous and $\mathcal{F}_0$ contains all $\mathcal{P}$-null sets). $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure $\mathcal{P}$. Denote by $L^2_{\mathcal{F}_0}([-\tau,0],\mathbb{R}^n)$ the family of all $\mathcal{F}_0-$measurable $C([-\tau,0],\mathbb{R}^n)-$valued random variables $\psi=\{\psi(s):s\in[-\tau,0]\}$ such that $\sup_{s\in[-\tau,0]}\mathbb{E}\{|\psi(s)|\}<\infty$. Matrices, if not explicitly specified, are assumed to have compatible dimensions.

2. Model description and preliminaries

In this paper, we consider the following CVNN with probabilistic time-varying delays:

$$\dot z(t)=-Cz(t)+Af(z(t))+Bf(z(t-\tau(t)))+J, \tag{1}$$

for $t\ge 0$, where $z(t)=(z_1(t),z_2(t),\dots,z_n(t))^T\in\mathbb{C}^n$, $z_i(t)$ is the state of the ith neuron at time t; $f(z(t))=(f_1(z_1(t)),f_2(z_2(t)),\dots,f_n(z_n(t)))^T\in\mathbb{C}^n$ and $f(z(t-\tau(t)))=(f_1(z_1(t-\tau(t))),f_2(z_2(t-\tau(t))),\dots,f_n(z_n(t-\tau(t))))^T\in\mathbb{C}^n$ are the vector-valued activation functions without and with time delays whose elements consist of complex-valued nonlinear functions; $\tau(t)$ corresponds to the transmission delay; $C=\mathrm{diag}\{c_1,c_2,\dots,c_n\}\in\mathbb{R}^{n\times n}$ is the self-feedback connection weight matrix, where $c_i>0$; $A\in\mathbb{C}^{n\times n}$ is the connection weight matrix, $B\in\mathbb{C}^{n\times n}$ is the delayed connection weight matrix; $J\in\mathbb{C}^n$ is the input vector.

For a complex-valued neural network, the main challenge is the choice of activation function. Any regular analytic function cannot be bounded unless it reduces to a constant. This is known as Liouville's theorem. That is to say, activation functions in complex-valued neural networks cannot be both bounded and analytic. Throughout this paper, we make the following assumptions:

Assumption 1. For any $i\in\{1,2,\dots,n\}$, $f_i(\cdot)$ is continuous and bounded and there exists a positive diagonal matrix $L=\mathrm{diag}\{l_1,l_2,\dots,l_n\}$ such that

$$|f_i(\alpha_1)-f_i(\alpha_2)|\le l_i|\alpha_1-\alpha_2| \tag{2}$$

for all $\alpha_1,\alpha_2\in\mathbb{C}$.

Assumption 2. $\tau(t)$ is bounded with $0\le\tau_1\le\tau(t)\le\tau_3$, and its probability distribution can be observed, i.e., suppose $\tau(t)$ takes values in $[\tau_1,\tau_2]$ or $(\tau_2,\tau_3]$ and $Prob\{\tau(t)\in[\tau_1,\tau_2]\}=\delta_0$, where $\tau_1\le\tau_2\le\tau_3$ and $0\le\delta_0\le 1$.

Remark 1. It is noted that the introduction of binary stochastic variable was first introduced in [20], and then successfully used in [21–29]. From Assumption 2, we know that δ_0 is dependent on the values of τ_1, τ_2 and τ_3, and $Prob\{\tau(t)\in(\tau_2,\tau_3]\}=1-\delta_0$.

In order to describe the probability distribution of the time delay, we define two sets

$$\Theta_1=\{t\,|\,\tau(t)\in[\tau_1,\tau_2]\},\quad \Theta_2=\{t\,|\,\tau(t)\in(\tau_2,\tau_3]\} \tag{3}$$

and introduce time-varying delays $\tau_1(t)$ and $\tau_2(t)$ such that

$$\tau(t)=\begin{cases}\tau_1(t), & t\in\Theta_1\\ \tau_2(t), & t\in\Theta_2.\end{cases} \tag{4}$$

It follows from (3) that $\Theta_1\cup\Theta_2=\mathbb{R}^+$ and $\Theta_1\cap\Theta_2=\varnothing$. From (4) it can be seen that $t\in\Theta_1$ implies the event $\tau(t)\in[\tau_1,\tau_2]$ occurs and $t\in\Theta_2$ implies that the event $\tau(t)\in(\tau_2,\tau_3]$ occurs.

Assumption 3. There are constants μ_1, μ_2, μ_3 and μ_4 such that $\mu_1\le\dot\tau_1(t)\le\mu_2$ and $\mu_3\le\dot\tau_2(t)\le\mu_4$.

Defining a stochastic variable as

$$\delta(t)=\begin{cases}1, & t\in\Theta_1\\ 0, & t\in\Theta_2,\end{cases} \tag{5}$$

then system (1) can be equivalently rewritten as

$$\dot z(t)=-Cz(t)+Af(z(t))+\delta(t)Bf(z(t-\tau_1(t)))+(1-\delta(t))Bf(z(t-\tau_2(t)))+J. \tag{6}$$

And the initial condition associated with model (6) is given by $z(s)=\phi(s)$, where $\phi(s)$ is continuously differential on $s\in[-\tau_3,0]$.

Remark 2. Under Assumption 2 and the definition of $\delta(t)$, it can be seen that $\delta(t)$ is a Bernoulli distributed white sequence with $Prob\{\delta(t)=1\}=Prob\{\tau(t)\in[\tau_1,\tau_2]\}=\mathbb{E}\{\delta(t)\}=\delta_0$ and $Prob\{\delta(t)=0\}=Prob\{\tau(t)\in(\tau_2,\tau_3]\}=1-\mathbb{E}\{\delta(t)\}=1-\delta_0$. Furthermore, it can be shown that $\mathbb{E}\{\delta^2(t)\}=\delta_0$, $\mathbb{E}\{(1-\delta(t))^2\}=1-\delta_0$ and $\mathbb{E}\{\delta(t)(1-\delta(t))\}=0$.

The following lemmas are useful in the derivation of the main result.