Neurocomputing ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

# Robust pole assignment in a specified union region using harmony search algorithm

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ARTICLE INFO

*Article history:*
Received 24 October 2014
Received in revised form
9 December 2014
Accepted 20 December 2014
Communicated by H. Zhang

*Keywords:*
Pole assignment
Robustness
Condition number
State feedback
Harmony search

ABSTRACT

This paper considers the problem of robust pole assignment for linear time-invariant systems by state feedback in a single circular region or a set of disjoint circular regions. A sufficient condition for the robust measure object is derived and it ensures the poles of the closed-loop system to remain within the specified union region when the perturbation or uncertainty appears. In order to get a set of poles and the corresponding state feedback controller which allow the system to have a maximum allowable perturbation or uncertainty for region pole constraint, the harmony search algorithm is employed to address the related optimization problem based on stochastic optimization approach. The simulation results demonstrate the effectiveness of the proposed approach.


## 1. Introduction

Stability is a minimum requirement for control systems in general. However, it is inevitable that model and parameter uncertainty exist in practical control systems, thus robust stability and robust performance are important for control system design. Robust pole assignment by state feedback is a basic design approach to achieve some desired robust stability and transient performance for the closed-loop system. It is known that when all state variables of a system are completely controllable and observable, the poles of the closed-loop system can be placed at any desired locations on the complex plane via proper state feedback law. Robust pole assignment by state feedback for linear time-invariant control system has been investigated by many researchers and various robustness measures and optimization approaches have been proposed, see e.g., [1–49]. Among these results, there are two approaches for studying robust pole assignment by state feedback for linear control systems.

The first approach considers the exact robust pole assignment with desired poles pre-specified in advance, see e.g., [1–31]. Kautsky et al. first proposed to minimize the spectral condition number of eigenvector matrix for state feedback robust pole assignment in [1], which has received a lot of attention due to the small number of arithmetic operations per iteration required by the numerical methods. Byers and Nash reformulated this method using Newton and truncated Newton iterations to minimize the Frobenius condition number for robust pole assignment in state feedback control systems in [2]. Later, gradient flow and neural network approaches to robust pole assignment were proposed in [6–11]. Recently, Le and Wang proposed a novel optimization approach based on recurrent neural networks for robust pole assignment in [12]. However, as robust pole assignment is a non-linear optimization problem, the twice differentiable and substantial gradient information of the optimization object function must be considered for most existing methods. In addition, it is difficult to place the desired poles at the exact locations due to the perturbation or uncertainty in engineering practice, and the robustness of the closed-loop system caused by the poles unconsidered for the proposed methods. In these cases, the applicable scopes are limited for the proposed methods.

Another approach considers region robust pole assignment and it focuses on placing the desired poles in a pre-specified region of the complex plane and ensuring the poles of the closed-loop system remain within the specified region when the system perturbation or uncertainty appears (see e.g., [32–49]). A method for pole assignment in a specified disk for linear feedback control systems based on the Riccati equation with norm-bounded uncertainty was studied in [33,39]. However, the locations of poles within the specified disk are dependent on the matrices of the Riccati equation in this method.

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http://dx.doi.org/10.1016/j.neucom.2014.12.072
0925-2312/

Chilali et al. summarized the methods of robust pole assignment in LMI regions in [41], in which the specified regions are assumed usually connected and often even convex for the LMIs descriptions. However, if there is a different response time or a specified damping ratio for the desired dynamic performances in practical control systems, the desired poles may be placed in different regions. Hence, the continuous convex constrained region considered in [41] cannot fully meet the needs of practical control system design. Later, Maamri et al. considered the problem of pole assignment in a set of disjoint circular regions using LMI method in [43]. However, in terms of the robustness of the system, the uncertainty or perturbation is required to satisfy the specified conditions in this study. As the perturbation or uncertainty is unknown in practical control systems, the LMI method considered is not universal in [43]. In addition, it is difficult to deal with nonlinear or non-convex constraints for several LMI methods. Using one LMI solution may lead to a very conservative controller design [31,50,51].

Harmony search (HS) algorithm is a meta-heuristic algorithm proposed by Geem et al. in 2001 [52] and it mimics a musician's improvisation process to find a global optimal solution or a near optimal solution determined by an objective via searching experience and effective exploration. Unlike the traditional optimization algorithms based on the Gradient and Newton's Methods, HS algorithm imposes fewer mathematical requirements and the derivative information is unnecessary [53]. Due to its simplicity, generality and flexibility and its lower parameter sensitivity, HS algorithm has been successfully applied to various types of engineering non-linear optimization problems, see e.g., [53–56].

In this paper, a novel approach is presented for the robust pole assignment by state feedback in a specified union region based on the harmony search algorithm. The specified union region considered may be a single circular region or a set of disjoint circular regions. Our design objective focuses on extending the idea of exact robust pole assignment to robust pole assignment in a specified union region for linear feedback control system and the closed-loop system with a better robustness for region pole constraint in some sense.

The remainder of this paper is organized as follows. In Section 2, a brief overview of the harmony search (HS) algorithm is presented. In Section 3, the problem of robust pole assignment in a specified union region by state feedback for linear feedback control system is discussed and a sufficient condition for robust region stable and robust performance optimization objective function is derived. Section 4 is based on the geometric descript for the position of the poles in the circular region, and the method of robust controller design is discussed in detail. In Section 5, the simulation of the application examples is considered to demonstrate the effectiveness of the proposed approach. Finally, a brief conclusion is given in Section 6.

*Notations*: In this paper, $\lambda(A)$ denotes the spectrum of matrix $A$; $\lambda_j(A)$ denotes the $j$th eigenvalue of matrix $A$; $\rho(A) \equiv \max|\lambda_j(A)|$ is the spectral radius of matrix $A$; $|A| = [|a_{ij}|]$ for a matrix $A = [a_{ij}]$; $\|\cdot\|$ denotes $\|\cdot\|_1$, $\|\cdot\|_2$ or $\|\cdot\|_\infty$; $I = \mathrm{diag}(1,1,\dots,1)_{n\times n}$; $A^T$ denotes the transpose of the matrix $A$.

**Lemma 1** (*Yang [57], Horn and Johnson [58]*).

(1) *If $A, B \in R^{n\times n}$ and $B \ge |A|$, then $\rho(B) \ge \rho(|A|) \ge \rho(A)$.*
(2) *Suppose that $A \in R^{n\times n}$, then $I - A$ is nonsingular if $\rho(A) < 1$.*

## 2. Harmony search (HS) algorithm

HS algorithm is based on musicians' search for a better state of harmony judged by the corresponding value of the objective function evaluation. The required parameters of the harmony search are as follows: harmony memory size (*HMS*) which is the number of solution vectors in harmony memory; harmony memory considering rate (*HMCR*); bandwidth (*bw*); pitch adjusting rate (*PAR*); and number of improvisations (*NI*). The optimization procedure of the HS algorithm consists of following steps:

*Step* 1: Initialize the optimization problem and algorithm parameters.

The optimization problem is defined as

$$\begin{aligned} &\text{Minimize} \quad f(x) \\ &\text{s.t.} \quad x_{jL} \le x_j \le x_{jU} \ (j = 1,2,\dots,N), \end{aligned} \tag{1}$$

where $f(x)$ is the object function; $x_{jL}$ and $x_{jU}$ are the lower and upper bounds for decision variables $x_j$, respectively; and $N$ is the number of design variables.

*Step* 2: Initialize the harmony memory.

Harmony memory (HM) contains a set of harmony vectors (solutions) whose sizes equal to *HMS*. Each component of harmony vector $x'$ is determined by the decision variables $x'_j (j = 1,2,\dots,N)$, where $x'_j$ is generated randomly by the following equation:

$$x'_j = x'_{jL} + (x'_{jU} - x'_{jL}) \times \mathrm{rand}(0,1), \tag{2}$$

where $x'_{jL}$ and $x'_{jU}$ are the lower bound and upper bound of the component $x'_j$ , respectively. Each harmony $x'$ represents a solution vector, and the objective value is evaluated according to the Eq. (1).

*Step* 3: Improvise a new harmony from the HM.

A new harmony vector $x^{new}$ is generated from the HM. This is called improvisation. The $j$th component of the new harmony vector, i.e., $x_j^{new}$ is determined by three rules: memory consideration, pitch adjustment and random selection. The procedure works as follows:

```
for each j ∈ [1, N] do
  if rand(0, 1) ≤ HMCR then
    x_j^new = x_j^i (i = 1, 2, …, HMS) %memory consideration
    if rand(0, 1) ≤ PAR then
        x_j^new = x_j^new ± rand(0, 1) × bw %pitch adjustment
    end if
  else
        x_j^new = x_jL + rand(0, 1) × (x_jU − x_jL) %random selection
    end if
  end for
```

where $x_j^{new}$ is the $j$th component of $x^{new}$, and $x_j^i$ is the $j$th component of $x^i$, and $bw$ is an arbitrary distance bandwidth.

*Step* 4: Update harmony memory.

If the fitness of the improvised harmony vector $x^{new}$ is better than that of the worst harmony $x^{worst}$, replace the worst harmony in the HM with $x^{new}$.

*Step* 5: Check the stopping criterion.

If the maximal iteration number (*NI*) is satisfied, computation is terminated. Otherwise, Steps 3 and 4 are repeated.

See Fig. 1 for the flow chart of the HS algorithm.

## 3. Robust pole assignment in a specified union region

In this section, we first introduce the problem of pole assignment by state feedback for linear feedback control system. Based on the problem of robust pole assignment by state feedback, we then discuss the problem of robust pole assignment in a specified union region via state feedback.

### 3.1. Pole assignment by state feedback

Consider a linear time-invariant control system as follows:

$$\dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0, \tag{3}$$