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A new probability model for insuring critical path problem with heuristic algorithm

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ABSTRACT

In order to obtain an adequate description of risk aversion for insuring critical path problem, this paper develops a new class of two-stage minimum risk problems. The first-stage objective function is to minimize the probability of total costs exceeding a predetermined threshold value, while the second-stage objective function is to maximize the insured task durations. For general task duration distributions, we adapt sample average approximation (SAA) method to probability objective function. The resulting SAA problem is a two-stage integer programming model, in which the analytical expression of second-stage value function is unavailable, we cannot solve it by conventional optimization algorithms. To avoid this difficulty, we design a new hybrid algorithm by combining dynamic programming method (DPM) and genotype-phenotype-neighborhood based binary particle swarm optimization (GPN-BPSO), where the DPM is employed to find the critical path in the second-stage programming problem. We conduct some numerical experiments via a critical path problem with 30 nodes and 42 arcs, and discuss the proposed risk averse model and the experimental results obtained by hybrid GPN-BPSO, hybrid genetic algorithm (GA) and hybrid BPSO. The computational results show that hybrid GPN-BPSO achieves the better performance than hybrid GA and hybrid BPSO, and the proposed critical path model is important for risk averse decision makers.

1. Introduction

In a complex project management problem, we often use a directed network graph to describe various tasks and the relationships among the tasks. In this framework, the arcs represent dependent tasks and the arc weights serve as associated task durations. Also, there exists a node 0 representing the start of the project and a node n representing its termination. A project can be considered completed if all its activities have been finished. An important theoretical result is that the minimum time to complete all the activities in the activity network equals to the length of the longest path from the source node to the destination node [1]. Thus, this path, called *critical path*, represents the sequence of activities, which will take the longest time to complete. Chen et al. [2] developed a polynomial time algorithm to find the critical path and analyzed the float of each arc in a time-constrained activity network. Guerriero and Talarico [3] proposed a general method to find the critical path in a deterministic activity-on-the-arc network, considering three different types of time constraints. Another area of research dealt with the stochastic nature of activity time. For example, Kelley [4,5] and Moehring [6] estimated the probability that a project would be completed by a given deadline if the duration for each activity is not known with certainty; Burt and Garman [7], Bowman [8] and Mitchell and Klastorin [9] treated mass uncertain information by heuristic-based and Monte Carlo simulation-based techniques, and Shen et al. [10] proposed expectation and chance-constrained models for insuring critical path problems and designed decomposition strategies to solve these models.

In this paper, we approach the insuring critical path problem from a new viewpoint. It is known that the appropriateness of expectation criterion for insuring critical path problem depends on the assumption that the insuring process can be repeated a great number of times, this implies by the law of large numbers that in the long run the average cost will be equal to the expected cost. But, this assumption will often not be justified and thus the expected cost may not be of much interest to risk averse decision makers. On the other hand, the optimal solution of the expected value problem may only assure the achievement of the corresponding expected cost with a relatively small probability. Consequently, the risk averse decision maker will not consider the solution of the expected value problem to be optimal. Instead, what may be desired is a solution ensuring a low probability of very large costs. These considerations lead us to adopt minimum risk criterion in insuring critical path problems. In the proposed

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risk averse two-stage stochastic insuring critical path problem, the first-stage objective function is to minimize the probability of total costs exceeding a predetermined threshold value, while the second-stage objective function is to maximize the insured task durations. For general task duration distributions, we adapt the SAA method to probability objective function and turn the original insuring critical path problem into its associated SAA one. This approximation method for chance constrained model and expected value model have been discussed in [11–13]. Since the resulting SAA model belongs to the class of NP-hard problems, we cannot solve it by conventional optimization algorithms. In this paper, we will employ evolutionary algorithms (EAs) to solve the resulting SAA critical path problem.

EAs are stochastic search methods that have been used in a variety of fields. Among existing EAs, GA [14] and PSO [15,16] are the well-known tools for solving complex optimization problems, and many modified and improved GA and PSO as well as their successful applications can be found in the literature. For example, Zeng et al. [17] proposed a dynamic chain-like agent GA for solving global numerical optimization problem; He and Tan [18] proposed a two-stage GA and applied it to automatic clustering; Lee et al. [19] modified and improved BPSO; Nanni and Lumini [20] proposed an efficient method based on PSO for finding a good set of prototypes; Qin and Liu [21,22] designed Monte Carlo simulation-based GAs to solve stochastic data envelopment analysis problems, and Liu et al. [23] solved stochastic portfolio selection problems by Monte Carlo simulation-based PSO algorithms. Motivated by the work mentioned above, this paper designs a new hybrid algorithm by combining DPM and GPN-BPSO, where DPM is employed to find the critical path in the second-stage programming problem. In our designed algorithm, we adopt the concept of genotype–phenotype in biology. The genotype means genetic messages carried by the individual's genes, and the phenotype denotes all the observable characteristics of an individual such as physical appearance and internal physiology. To demonstrate the effectiveness of the proposed method, we conduct some numerical experiments via a critical path problem with 30 nodes and 42 arcs. We first solve our critical path problem by hybrid GPN-BPSO, then compare its solution results with those obtained by hybrid GA and hybrid BPSO. We also discuss the difference between the proposed risk averse insuring critical path model and the traditional risk neural model via numerical experiments.

The remainder of this paper is organized as follows: Section 2 presents a new class of risk averse two-stage stochastic insuring critical path problems. In Section 3, we adapt the SAA method to probability objective function, and turn the original insuring critical path problem into its associated SAA one, which can be reformulated as a two-stage integer programming model by introducing additional binary variables. To solve the resulting SAA critical path problem, Section 4 designs a new hybrid algorithm by integrating DPM and GPN-BPSO. Section 5 provides one critical path problem with 30 nodes and 42 arcs and performs some numerical experiments to demonstrate the effectiveness of the designed hybrid GPN-BPSO. Section 6 gives detailed discussions about the proposed insuring critical path model and the experimental results. Finally, Section 7 gives the conclusions.

2. Formulation of risk averse two-stage stochastic insuring critical path problem

In this section, we will construct a risk averse two-stage stochastic optimization model for insuring critical path problem. For this purpose, we adopt the following notations to describe our problem.

Indices:

i: index of nodes, $i \in N$;
j: index of nodes, $j \in N$.

Parameters:

$N=\{0,1,\ldots,n\}$: the set of nodes in the network;
A: the set of arcs in the network, $A \subset N \times N$, where A is topologically ordered such that $(i,j) \in A$ only if $i<j$;
$G(N,A)$: the directed graph representing the tasks to be completed in a complex project;
$FS(i)=\{j|(i,j)\in A\}$: the set of nodes adjacent from node i, $\forall i \in N$;
$RS(i)=\{j|(j,i)\in A\}$: the set of nodes adjacent to node i, $\forall i \in N$;
Ω: the set of possible scenarios;
ω: a scenario of Ω;
c_{ij}: the cost of insuring arc $(i,j)\in A$;
d_{ij}^{ω}: an uninsured task duration of arc $(i,j)\in A$ in scenario ω;
g_{ij}^{ω}: an insured task duration of arc $(i,j)\in A$ in scenario ω;
Θ: a nondecreasing function of task completion time that penalizes the critical path length in the second stage for each scenario ω;
ξ: the random vector obtained by piecing together all random task durations in the network;
$\overline{\varphi}$: a predetermined maximum allowable cost.

Decision variables:

x_{ij}: 1 if arc (i,j) is insured, and 0 otherwise;
x: a decision vector (x_{ij}) in $\{0,1\}^{|A|}$ with $|A|$ being the number of arcs in the network;
y_{ij}^{ω}: 1 if arc (i,j) is part of one identified critical path in scenario ω, and 0 otherwise.

The second-stage objective function:

The second-stage objective function is to maximize the sum of the insured task durations:

$$\max \sum_{(i,j)\in A} (d_{ij}^{\omega}-(d_{ij}^{\omega}-g_{ij}^{\omega})x_{ij})y_{ij}^{\omega}.$$

The second-stage constraints:

The first constraint imposes a single-assignment rule:

$$\sum_{j\in FS(0)} y_{0j}^{\omega}=1.$$

The second constraint enforces flow-balance constraints for critical path contiguity:

$$\sum_{j\in FS(i)} y_{ij}^{\omega}-\sum_{l\in RS(i)} y_{li}^{\omega}=0,\ \forall i\in N\backslash\{0,n\}.$$

The third constraint bounds a binary decision variable:

$$y_{ij}^{\omega}=\begin{cases}1 & \text{if arc } (i,j) \text{ is part of an identified critical path in scenario } \omega\\ 0 & \text{otherwise.}\end{cases}$$

Hence, the second-stage programming problem can be built as follows:

$$\begin{cases}\max & \sum_{(i,j)\in A} (d_{ij}^{\omega}-(d_{ij}^{\omega}-g_{ij}^{\omega})x_{ij})y_{ij}^{\omega}\\ \text{subject to}: & \sum_{j\in FS(0)} y_{0j}^{\omega}=1\\ & \sum_{j\in FS(i)} y_{ij}^{\omega}-\sum_{l\in RS(i)} y_{li}^{\omega}=0,\ \forall i\in N\backslash\{0,n\}\\ & y_{ij}^{\omega}\in\{0,1\},\quad \forall (i,j)\in A.\end{cases} \tag{1}$$