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Adaptive critic design-based robust neural network control for nonlinear distributed parameter systems with unknown dynamics

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ABSTRACT

In this paper, an adaptive critic design (ACD)-based robust on-line neural network control design is developed for a class of parabolic partial differential equation (PDE) systems with unknown nonlinear dynamics. First, the Galerkin method is applied to the parabolic PDE system to derive a finite-dimensional slow one and an infinite-dimensional stable fast subsystem. The obtained slow system is an ordinary differential equation (ODE) system with unknown nonlinearities, which accurately describes the dynamics of the slow modes of the PDE system. Then, a novel ACD-based robust optimal control scheme is proposed for the resulting nonlinear slow system with unknown dynamics. An action neural network (NN) is employed to approximate all the derived unknown nonlinear terms and a robust control term is further developed to attenuate the NN reconstruction errors and disturbances. Especially, by developing novel critic signals and Lyapunov function candidate, together with the adaptive bounding technique, no a prior knowledge for the bounds of the disturbance term, the NN ideal weights of action NN and critic NN and the NN reconstruction errors is required. Finally, simulation results demonstrate the effectiveness of the proposed robust optimal control scheme.

1. Introduction

In practice, most physical systems are inherently distributed in time and space, such as biodynamics, chemical engineering, and mechanical systems related to heat flows, fluid flow, or flexible structure. In the past, most physical systems were modeled by ordinary differential equations (ODEs) in order to simplify and systematically solve control-design problems. However, if the variation of the system which depends on the space is considered, it is not accurate to model the physical systems as ODEs. Therefore, the distributed parameter system (DPS) described by partial differential equations (PDE) attracts more and more attentions. Since the outputs, inputs, and process states and the relevant parameters of DPS may vary temporally as well as spatially, it is more suitable to model the spatiotemporal dynamic systems with DPS.

In general, DPS is described by a set of PDEs with mixed or homogeneous boundary conditions. However, due to the spatially distributed nature and dynamic complexity, it is difficult to design controller for DPS. Fortunately, recent research [1–4] shows that the dynamics of the parabolic PDEs can be described approximately in a group of low-order ordinary differential equations. In [3], the author proposes a simple but effective modeling method for DPS by integrating the spectral method with neural networks. In [4], the authors stabilize DPS via the Galerkin method and the geometric control. In [5], the K–L method is employed to model the distributed parameter system, where a class of DPS modeled by parabolic PDE is considered. The eigenspectrum of parabolic spatial differential operator can be separated into a finite-dimensional slow one and an infinite stable fast complement. Based on Galerkin's method, the proposed lower order ODE systems can sufficiently describe dominant dynamics of DPS, and thus can be used as the basis of controller design while the fast system is stabilized.

However, when a plant is confronted in practice, whatever it is modeled as deterministic systems or stochastic systems [29,30], guaranteeing the stability of equilibrium point is just the basic requirement and how to obtain the optimal performance index is the main focus. Over the past decades, the optimal control theory of PDE systems has been early presented by Butkovskiy [6] and Lions [7], and more theoretical results can be found in [8,9]. Meanwhile, the optimal control problem of PDE systems has also been well studied in engineering applications based on the minimization of linear quadratic (LQ) performance indices. However, most mentioned results require that the nonlinear functions of parabolic PDE systems are completely known. As far as we know, till now there is not optimal control results for DPS with unknown nonlinear dynamics.

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On the other hand, in recent years, in order to obtain approximate solutions of the Hamilton–Jacobi–Bellman (HJB) equation, adaptive/approximate dynamic programming (ADP) algorithm was proposed in [13,21,14] as the most potential way. ADP combines adaptive critic design (ACD), reinforcement learning technique with dynamic programming, which have gained much attention from a lot of researchers, cf. [15–20,22]. As one of ADP serials, ACD has held great intuitive appeal and has attracted considerable attention in the past few years. In general, ACD involves a critic neural network (NN) and an action NN, where the critic NN evaluates the performance of current control and generates a critic signal to update the control action for performance improvement, and the action NN provides the control input to the system. There are several works dealing with the application of ACD methods for nonlinear systems in both discrete and continuous time. For example, some variants of reinforcement learning neural-network-based controllers were proposed with actor-critic architectures in [23,24] for discrete-time nonlinear systems. For continuous-time nonlinear system, several studies have been made about ACD methods based on neural network [25,26] or fuzzy system [27,28], respectively.

It should be noted that all of the aforementioned works are limited to affine nonlinear systems. Although ACD algorithms have made large progress in the optimal control field, to the best of our knowledge, few result is available for more complicate distributed parameter systems. Especially, there is still no result to solve the robust optimal control problem for PDE systems with partial unknown nonlinear dynamics. Therefore, in this paper, a novel ACD-based robust NN controller is proposed for a class of PDE system with unknown dynamics. Inspired by the works of [10–12], we employ the Galerkin method to convert the dynamics of DPS into a group of low-order functional ordinary differential equations, i.e., a low-dimensional slow system. Then based on [27], we develop a novel critic signal to evaluate the performance of the controller, and meanwhile a novel Lyapunov function is proposed to guarantee the stability of the presented control scheme. Specifically, the developed critic signal is comprised of two parts, i.e., the primary critic signal and the second critic signal. The primary critic signal is the function of filtered tracking error and the second critic signal is the output of the critic NN. Meanwhile, an action NN is employed to approximate the derived all the unknown term instead of the usual unknown nonlinear function. Additionally, a robust term is developed to attenuate the NN reconstruction errors introduced by action NN and critic NN.

Briefly speaking, the main contributions of this paper can be summarized as follows:

(1) Compared with Reference [11], we do not need to train the NN by BP algorithm off-line and no NN approximate error need to be estimated in advance before solving the controller gain. In this paper, it is the first time to develop the robust on-line optimal control scheme for DPS with unknown nonlinear dynamics, where the robust stability and optimality synthesis of the closed-loop system is simultaneously performed by Lyapunov method. The NN approximate error and disturbance are considered without knowing the boundedness, whereas in [11] the NN approximate error is ignored.

(2) Compared with Reference [12], we consider the nonlinear PDE systems with unknown dynamics, and the control law is directly derived based on an adaptive control principle rather than alternant iteration of control laws and cost functions as in [12]. Moreover, the computation of GHJB-like equation at each iteration is not required in this paper, which shorten the computation time greatly.

(3) By developing a novel critic signal and novel Lyapunov function, along with the new parameter adjusting method and adaptive bounding technique, the uniformly ultimate boundedness of all signals in the closed-loop system is proved. Further,the assumption on the bounds of disturbance term, the ideal weights of action NN and critic NN, and the NN reconstruction errors has been removed.

The paper is organized as follows. In Section 2, the problem statement is given. The ACD-based robust NN controller design is presented in Section 3. In Section 4, the Lyapunov stability analysis for the whole closed-loop system is given. The simulation results are presented in Section 5, where the satisfactory performance of the proposed approach is shown. The conclusion is drawn in Section 6.

2. Problem formulation and preliminaries

Consider the following nonlinear parabolic PDE systems in one spatial dimension with a state-space representation of the form

$$\frac{\partial x}{\partial t}=E_1\frac{\partial x}{\partial z}+E_2\frac{\partial^2 x}{\partial z^2}+f(x)+k_u b(z)u(t)+d(t), \tag{1}$$

subject to the boundary conditions

$$P_1 x(\alpha_1,t)+Q_1\frac{\partial x}{\partial z}(\alpha_1,t)=h_1,$$
$$P_2 x(\alpha_2,t)+Q_2\frac{\partial x}{\partial z}(\alpha_2,t)=h_2, \tag{2}$$

and the initial condition

$$x(z,0)=x_0(z), \tag{3}$$

where $x(z,t)=[x_1(z,t),x_2(z,t),\dots,x_n(z,t)]^T$ is the vector of state variables, $z\in[\alpha_1,\alpha_2]$ is the spatial coordinate, $t>0$ is the time, and $u(t)\in\mathbb{R}^p$ is the applied force to be designed, which is provided by p-point force actuators. $\partial x/\partial z$ and $\partial^2 x/\partial z^2$ are the first-order and second-order spatial derivatives of x, respectively. $f(x)$ is an unknown nonlinear vector function, which is locally Lipschitz continuous and satisfies $f(0)=0$. $x_0(z)$ is the initial condition and k_u is a constant vector. $b(z)=[b_1(z),b_2(z),\dots,b_p(z)]$ is a known smooth vector function of z, where $b_i(z)$ describes how the control action u_i is distributed in the interval $[\alpha_1,\alpha_2]$, E_1,E_2,P_1,Q_1,P_2 and Q_2 are constant matrices, h_1,h_2 are column vectors. $d(t)$ is a bounded disturbance term, but the boundedness value is unknown.

In this paper, we aim to seek the optimal controller for this DPS (1)–(3). However, due to the spatially distributed nature and the existence of unknown nonlinearities, it does not allow one to directly carry out the control design based on PDEs (1)–(3). For model-based synthesis method and real-time controller design, a low-dimensional NN-based ODE system is preferred. Therefore, in the next part, the Galerkin method is used to reduce the model (1)–(3) to a low-dimensional ODE system with unknown nonlinearities first.

Inspired by the work of [10,11], a parabolic PDE involves spatial differential operators whose spectrum can be partitioned into a finite-dimensional and an infinite-dimensional complement, i.e., the slow and fast components. This implies that the dynamical behavior of such a system can be approximately described by a finite-dimensional ODE system that captures the dynamics of the dominant modes of the PDE. Generally, for such a parabolic PDE system, if the boundary conditions are homogenous, it is feasible to use the eigenfunctions of a spatial differential operator to derive a low-order ODE system.

Define $\mathbb{S}$ as a Hilbert space of 1-D functions satisfying the boundary conditions (2), with inner product and norm

$$\langle\varpi_1,\varpi_2\rangle_p=\int_{\alpha_1}^{\alpha_2}\langle\varpi_1,\varpi_2\rangle\,dz \tag{4}$$

and

$$\|\varpi_1\|_2=\langle\varpi_1,\varpi_1\rangle_p^{1/2}, \tag{5}$$

where ϖ_1,ϖ_2 are two elements of $\mathbb{S}$ and $\langle\cdot,\cdot\rangle$ denotes the standard inner product in $\mathbb{R}^n$. Define the spatial operator $\mathfrak{T}$ as

$$\mathfrak{T}x=E_1\frac{\partial x}{\partial z}+E_2\frac{\partial^2 x}{\partial z^2}, \tag{6}$$