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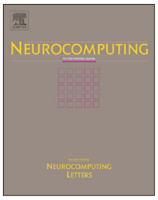

Probabilistic optimal power flow for power systems considering wind uncertainty and load correlation

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A B S T R A C T

Considering wind uncertainty and load correlation, this paper is concerned with the probabilistic optimal power flow (POPF) calculation. The POPF model for wind-integration power system is firstly proposed. Two kinds of schemes in point estimate method, $2m$ and $2m+1$ scheme, are then employed to solve the POPF. Moreover, the correlation samples of nodal injections are generated by the Cholesky decomposition method, and the path following interior point method is employed to solve the deterministic optimal power flow calculation. Finally, the proposed method is tested on the modified 5-bus and IEEE 30-bus test system. Simulation results show that $2m+1$ scheme is feasible and effective to solve the POPF for wind-integration power system. Also, it is seen that correlated loads affect the POPF results, and the POPF with load correlation would reflect the system operation more accurately.

1. Introduction

Wind energy, as a renewable green energy, is of great significance to alleviate the worldwide energy crisis. With more and more wind power integrated into the power system, wind power has been an important part of the construction of strong smart grid. So it is so urgent for experts and researchers to study the optimal operation mode for wind-integration power system due to the characteristics of wind as randomness, intermittent, and fluctuation. The probabilistic optimal power flow (POPF) is usually taken as an analytical tool to not only assess the system economy in steady state operation, but also analyze economic behaviors of the power producer and customer in the power market environment.

To conduct optimization between economic and safety, optimal power flow (OPF) calculation has been widely used in power system planning and operating. Based on power flow equations, OPF belongs to a multi-constraint and non-linear programming problem [1]. Different from the deterministic OPF, POPF can consider the randomness and fluctuation of nodal injections and wind speed and can obtain more accurate results.

The existing probabilistic methods applied to power systems include Monte Carlo simulation (MCS) [2,3], point estimate method [4–6], first order second moment method [7,8], cumulant method [9,10], and convolution techniques and its extension, etc. [11,12]. The probability description of state voltage and branch power flow can be obtained accurately by MCS [2,3]; however, it usually consumes large computation effort. The point estimate method [4–6] is widely applied to the probability distribution fitting of the OPF solution, which is based on the deterministic OPF calculation and can calculate the statistical moment of the quantity of state efficiently. The main idea of the first order second moment method [7,8] and cumulant method [9,10] is that the input and output random variables are expressed as its mean and fluctuation parts. Then, the Taylor expansion at the mean value of input random variables of the power flow equations is made, and the approximate linear relationship between the fluctuation parts of output and input random variables is obtained. Convolution techniques and its extension are used for probabilistic load flow analysis [11,12]. However, the literatures above mentioned, not considering the correlation between random variables such as load and generation power, produce the impractical probabilistic results.

To solve the probabilistic power flow problem with correlated parameter, the covariance matrix transformation technique [13] is combined into the two point estimate method with nodal injection mutual correlation. The method firstly determines the covariance matrix eigenvalues and eigenvectors of nodal injections. To compute the probabilistic load flow, the original correlated random variables are then transformed to a set of statistically independent random variables using orthogonal transformation. Considering spatially correlated power sources and loads in [14], a probabilistic power flow model is constructed, which is then solved using an extended point estimate method. The orthogonal transformation was also used to obtain the sample of input

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variables to solve the correlated random variables. The multivariate dependent random numbers with any given distribution are generated in [15,16]. Based on the inverse transformation of a uniform distribution, the dependence is achieved by the correlation matrix of a multivariate normal variable.

Motivated by the above observations, this paper investigates the POPF calculation for wind-integration power system with correlated loads and generation powers. Firstly, the POPF model for wind-integration power system is proposed. Next, the correlation load samples are generated by the Cholesky decomposition method. Based on the point estimate method, two kinds of different schemes (i.e., $2m$ and $2m+1$) and Monte Carlo Simulation are then used to solve the POPF with correlated loads. Furthermore, the path following interior point method is employed to solve the deterministic OPF. Finally, the POPF with correlated loads are made on the improved 5-bus and IEEE 30-bus test system to analyze the impacts of correlated loads on the power system operation.

The rest of paper is organized as follows. Section 2 describes the POPF model for wind-integration power system. A brief introduction of the Cholesky decomposition method is presented in Section 3. An overview of the point estimate method is described, and the complete algorithm of POPF problem for wind-integration power systems with correlated loads is presented in Section 4. Simulation results on the improved 5-bus and IEEE 30-bus test system are shown in Section 5, followed by the conclusions in Section 6.

2. Probabilistic optimal power flow model for wind-integration power system

2.1. Wind turbine model

In the estimation of long-term wind speed, it is usually modeled as follows [17]:

$$\varphi(v)=\frac{k}{c}\left(\frac{v}{c}\right)^{k-1}\exp\left[-\left(\frac{v}{c}\right)^{k}\right](k>0,v>0,c>1) \tag{1}$$

where v represents wind speed, and $\varphi(v)$ is the Weibull probability density function with the shape parameter k and the scale parameter c.

The wind turbine model can be expressed as follows:

$$P_m=\frac{1}{2}\rho A v^3 C_p \tag{2}$$

where ρ is the density of air (Kg/m^3), A is the area swept out by the turbine blades (m^2), v is the wind speed (m/s) and C_p is the dimensionless power coefficient. C_p can be expressed as a function of the blade tip speed ratio and be obtained by interpolation method.

As the random variations of wind speed with time, the corresponding outputs power of wind turbines also change. Therefore, the wind speed samples can be generated by simulating (1), and the corresponding wind power outputs can be calculated using (2).

2.2. Induction wind generator model

The induction generator's equivalent circuit can be simplified into $\Gamma-type$ equivalent circuit when induction generator is used in wind farm [18]. In short, the real power injected into grid and the reactive power absorbed from the grid generated by induction wind generator can be expressed as

$$P_e=-\frac{U^2 r_2/s}{(r_2/s)^2+x_k^2} \tag{3}$$

$$Q_e=-\frac{r_2^2+x_k(x_k+x_m)s^2}{r_2 x_m s}P_e \tag{4}$$

where x_1 is the stator reactance, x_2 is the rotor reactance, r_2 is the rotor resistance, $x_k=x_1+x_2$, x_m is the magnetizing reactance, s is the slip of induction machine, and U is the generator voltage, respectively. It can be seen from (3) and (4) that the reactive power Q_e absorbed by wind power generator has close relationship to the voltage U and the slip s for the certain real power P_e. The powers P_e and Q_e are random variation.

2.3. POPF model for wind-integration power system

Considering an N-bus system, the objective function of the POPF including wind farms is formulated as the minimization of the total fuel cost for conventional generation

$$\min \sum_{i\in S_G}(a_{2i}P_{Gi}^2+a_{1i}P_{Gi}+a_{0i}) \tag{5}$$

where S_G is the set of power generation, P_{Gi} is the output of conventional generator's real power in ith generator, a_{2i}, a_{1i} and a_{0i} are the generation cost coefficients, respectively. The equality constraints of the POPF including wind farms are the power flow equations in the rectangular coordinates. For the corresponding conventional node, the power flow equations are as follows:

$$\begin{cases} P_{Gi}-P_{Di}-P_i(e,f,t)=0 \\ Q_{Gi}-Q_{Di}-Q_i(e,f,t)=0\end{cases}, \quad i=1,\ldots,N-1, i\neq slack \tag{6}$$

where Q_{Gi} is output of reactive power, P_{Di} and Q_{Di} are real and reactive load, node powers P_i and Q_i are the function of the real part e and imaginary part f of node voltage and the ratio t, $slack$ is the slack node, and N_w is the set of the buses connected with wind farm, respectively. Based on probabilistic model in wind power generation system above mentioned, if wind farm is connected at the ith $(i\in N_w)$ bus, the corresponding power flow equations for wind-integration power system are given by

$$\begin{cases} P_{ei}(e_i,f_i,s_i)-P_{Di}-P_i(e,f,t)=0 \\ Q_{ei}(e_i,f_i,s_i)-Q_{Di}-Q_i(e,f,t)=0, \quad i\in N_w \\ P_{mi}-P_{ei}(e_i,f_i,s_i)=0 \end{cases} \tag{7}$$

where P_{mi} and P_{ei} are the mechanical power of wind turbine and the electrical power of wind generator at the ith bus, respectively. Note that the uncertainties of wind speed and load are combined, so the mechanical power, the real power injected into grid and reactive power absorbed from grid generated by wind power generator are described as a probabilistic model. Real and reactive power generations, ratios, voltage amplitudes and line currents are limited due to equipment and system constraints

$$\begin{cases} P_{Gi}^{\min}\le P_{Gi}\le P_{Gi}^{\max} & i\in S_G \\ Q_{Gi}^{\min}\le Q_{Gi}\le Q_{Gi}^{\max} & i\in S_G \\ t_{ij}^{\min}\le t_{ij}\le t_{ij}^{\max} & (i,j)\in S_T \\ (V_i^2)^{\min}\le(e_i^2+f_i^2)\le(V_i^2)^{\max} & i\in S_N \\ I_{ij}^2\le(I_{ij}^2)^{\max} & (i,j)\in S_L \end{cases} \tag{8}$$

where S_T, S_N and S_L are the set of the transformers, the system nodes and the restricted line, respectively.

In short, the above introduced POPF model can be further expressed as the following nonlinear programming problem:

$$\begin{aligned} \min \quad & f(x) \\ s.t. \quad & h(x)=0 \\ & g(x)\ge 0 \end{aligned} \tag{9}$$

where $x=[P_G^T,Q_G^T,t^T,f^T,e^T,s^T]^T\in R^n$ is the vector of system control variables and state variables; $f:R^n\to R$ is the objective function;