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Letters

An improved criterion for stability and attractability of memristive neural networks with time-varying delays

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ABSTRACT

Memristive neural networks are a novel topic in the design and construction of brain-like circuitry system. This paper addresses a challenging problem: How to derive some less conservative theoretical results on the stability and attractability for memristive neural networks? In this paper, a new comparison method and segmentation method of state space are developed. A succinct criterion is provided to ascertain the global exponential stability and the estimate for location of equilibrium point. The obtained criterion is the improvement and extension of the existing results in the literature. The applicability of the proposed framework can be extended to general memristive neurodynamic systems.

1. Introduction

Since the physical implementation of nanoscale memristor in 2008, extensive studies have been devoted to investigate memristive neural networks [1–17]. Memristive neural networks show many advantageous features for memory design such as non-volatility, low-power and good scalability [1,2,6–9]. Different from traditional electronic neural networks, the nonlinear properties of memristive neural networks are extremely complex [3–5,10–17]. One possible reason for the complexity of memristive neural networks may lie in the threshold sensitive memristor with a nonlinear drift effect. Meanwhile, a myriad of potential memristor applications calls for a thorough understanding of the nonlinear dynamic behaviors of memristive neural networks, in order to facilitate circuit analysis and design [12–15]. Further research on the dynamic mechanism of memristive neural networks will contribute to develop many physical memristive circuits and nonlinear circuit theories.

After development over several decades, numerous inspirational works on various kinds of nonlinear systems have been reported, see [18–46]. Whereas, in view of the memristor nonlinearity, a memristive neural network is a switched network cluster [3–5,10–17]. For a long time, the properties concerned switched network cluster have not received considerable attention. Obviously, the study on the switched network cluster is a more difficult and challenging problem. One of the main reasons is that the classical Lyapunov functional method is invalid, due to a great deal of switched effects. Whereupon some scholars are concerned about the multiple Lyapunov functional method. Generally, the multiple Lyapunov functional method is a classical but powerful tool for studying the nonlinear dynamics of hybrid systems. However, there are no general rules for constructing multiple Lyapunov functions. Even if the multiple Lyapunov functions are constructed, unfortunately, the conventional multiple Lyapunov functional method might not work very well [11–13,16,17], since a memristive neurodynamic system consists of too many subsystems. As a results, the resulting conditions are over-conservative. Therefore, the qualitative analysis of memristive neurodynamic systems deserves in-depth investigation.

In addition, most of the results of existing research are concerned with the criteria of global stability of memristive neural networks [3,10–12,16,17]. However, there are only a couple of works considering the related attractability [5,13,15], i.e., the location of the equilibrium point. In many real-world applications, it is necessary or desirable to estimate the location of equilibria, for example pattern memory analysis and goal programming [13,15]. It is worth considering the problem of the location of equilibria.

It is worth pointing out that the comparison method is an ideal way to deal with the complex nonlinear system. Whether we can

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introduce comparison method into switched network cluster, especially memristive neurodynamic systems? This is an open problem, which is ignored in the existing references. In fact, the Lyapunov functional method is also a primitive comparison method, i.e., the system states are compared with the contour curves of the Lyapunov function. Given the memristor nonlinearity, several efforts must be made to address the issue of some new comparison methods for memristive neurodynamic systems, in order to allow the memristor to be readily used as an analog memory element.

Motivated by the above-stated problems, in this paper, within the framework of the differential inclusions theory, a new comparison method is developed via the segmentation of state space. Based on the new method, an improved criterion for the stability and the attractability of memristive neural networks with time-varying delays has also been obtained. Roughly stated, the main advantages of this paper include the following four points:

(1) It should be noted that the stability of memristive neural networks has been studied in the literature [3,10–12,16,17]. Because of the similarity of structure, it will be interesting and challenging to develop a universal framework to study the memristive neural networks. Whether can we propose an effective approach to investigate the general memristive neural networks in a universal framework? This paper gives an affirmative answer on the basis of the comparison method.

(2) Most of the existing stability conditions are independent of external inputs. Because the dynamic evolution of memristive neural networks is evoked by external inputs and other network parameters, meanwhile, the locations of equilibria of memristive neural networks depend also on the external inputs, so the stability conditions independent of external inputs are over-conservative. In this paper, it is shown that global exponential stability of memristive neural networks can be achieved when the external inputs are reasonably selected.

(3) The improved conditions without ignoring location of equilibrium point in stability analysis may enhance the essential property of system. In contrast to the results in [3,10,11,16,17], the obtained results have considered the location of equilibrium point in stability analysis, which decreases the conservativeness of the theoretical results.

(4) Some comparisons are also given to show the superiority and differences of the resulting conditions over the previous results in [3,10,11,16,17]. Clearly, the proposed results are new and can be checked easily.

This paper is structured as follows. In Section 2, we give a short introduction to the problem formulation and preliminaries. In Section 3, we present the theoretic analysis framework. In Section 4, we discuss the validity of the proposed framework through some numerical examples. We finally conclude the study in Section 5.

2. Preliminaries

Consider a class of memristive neural networks with time-varying delays described by the following equations: for $i=1,2,\dots,n$,

$$\dot{x}_i(t) = -d_i x_i(t) + \sum_{j=1}^{n} a_{ij}(x_i(t)) f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(x_i(t)) f_j(x_j(t-\tau_{ij}(t))) + u_i, \tag{1}$$

where $x_i(t)$ is the voltage of the capacitor $\mathbf{C}_i$, $d_i>0$ is the self-inhibition, $\tau_{ij}(t)$ is the time-varying delay that satisfies $0 \le \tau_{ij}(t) \le \tau$ ($\tau \ge 0$ is a constant), $f_i(\chi) = (|\chi+1| - |\chi-1|)/2$ is the feedback function, u_i denotes the external input, $a_{ij}(x_i(t))$ and $b_{ij}(x_i(t))$ represent the memristor-based weights, and

$$a_{ij}(x_i(t)) = \frac{\mathbf{W}_{ij}}{\mathbf{C}_i} \times \text{sgin}_{ij}, \quad b_{ij}(x_i(t)) = \frac{\mathbf{M}_{ij}}{\mathbf{C}_i} \times \text{sgin}_{ij},$$

$$\text{sgin}_{ij} = \begin{cases} 1, & i \ne j, \\ -1, & i = j, \end{cases}$$

in which $\mathbf{W}_{ij}$ and $\mathbf{M}_{ij}$ denote the memductances of memristors $\mathbf{R}_{ij}$ and $\mathbf{F}_{ij}$, respectively. Here, $\mathbf{R}_{ij}$ represents the memristor across the feedback function $f_j(x_j(t))$ and $x_j(t)$, and $\mathbf{F}_{ij}$ represents the memristor across the feedback function $f_j(x_j(t-\tau_{ij}(t)))$ and $x_j(t)$.

According to the pinched hysteretic loops of ideal memristors, let

$$a_{ij}(x_i(t)) = \begin{cases} \hat{a}_{ij}, & x_i(t) > 0, \\ \hat{a}_{ij} \text{ or } \check{a}_{ij}, & x_i(t) = 0, \\ \check{a}_{ij}, & x_i(t) < 0, \end{cases} \tag{2}$$

$$b_{ij}(x_i(t)) = \begin{cases} \hat{b}_{ij}, & x_i(t) > 0, \\ \hat{b}_{ij} \text{ or } \check{b}_{ij}, & x_i(t) = 0, \\ \check{b}_{ij}, & x_i(t) < 0, \end{cases} \tag{3}$$

for $i,j=1,2,\dots,n$, where $\hat{a}_{ij}$, $\check{a}_{ij}$, $\hat{b}_{ij}$ and $\check{b}_{ij}$ are constants.

The initial condition of system (1) is assumed to be

$$x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T = \phi(t) = (\phi_1(t), \phi_2(t), \dots, \phi_n(t))^T, \quad t_0 - \tau \le t \le t_0, \tag{4}$$

where $\phi_i(t) \in \mathcal{C}([t_0-\tau, t_0], \mathfrak{R})$, $i=1,2,\dots,n$.

Remark 1. As in [3–5,10–17], from a systems-theoretic point of view, system (1) is essentially a switched network cluster, then we need to establish a solution for switched network cluster in the framework of nonsmooth analysis. For this purpose, the theory of Filippov is employed in this paper.

Remark 2. The uncertainty in system (1) is different from the robustness involved in some conventional nonlinear systems. The uncertainty of system (1) is state-dependent nonlinearity to be exact. Whereas, the robustness of conventional nonlinear systems is normally taken to mean the unavoidable modeling errors, external disturbance and parameter fluctuation.

Throughout this paper, solutions of all the systems considered in the following are intended in Filippov's sense. $K(\mathcal{P})$ denotes the closure of the convex hull of set $\mathcal{P}$ and $\text{co}\{\tilde{\Pi}, \widehat{\Pi}\}$ denotes the closure of the convex hull generated by real numbers $\tilde{\Pi}$ and $\widehat{\Pi}$. Let $\overline{a}_{ij} = \max\{\hat{a}_{ij}, \check{a}_{ij}\}$, $\underline{a}_{ij} = \min\{\hat{a}_{ij}, \check{a}_{ij}\}$, $\overline{b}_{ij} = \max\{\hat{b}_{ij}, \check{b}_{ij}\}$, $\underline{b}_{ij} = \min\{\hat{b}_{ij}, \check{b}_{ij}\}$, $\tilde{a}_{ij} = \max\{|\hat{a}_{ij}|, |\check{a}_{ij}|\}$, and $\tilde{b}_{ij} = \max\{|\hat{b}_{ij}|, |\check{b}_{ij}|\}$, for $i,j=1,2,\dots,n$.

By the theory of differential inclusions, from (1), for $i=1,2,\dots,n$,

$$\dot{x}_i(t) \in -d_i x_i(t) + \sum_{j=1}^{n} K(a_{ij}(x_i(t))) f_j(x_j(t)) + \sum_{j=1}^{n} K(b_{ij}(x_i(t))) f_j(x_j(t-\tau_{ij}(t))) + u_i, \tag{5}$$

where the set-valued maps

$$K(a_{ij}(x_i(t))) = \begin{cases} \hat{a}_{ij}, & x_i(t) > 0, \\ \text{co}\{\hat{a}_{ij}, \check{a}_{ij}\}, & x_i(t) = 0, \\ \check{a}_{ij}, & x_i(t) < 0, \end{cases} \tag{6}$$

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