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Letters

Nonlinear $\mathcal{H}_\infty$ observer design for one-sided Lipschitz systems

Wei Zhang [a], Housheng Su [b,*], Shengchao Su [a,c], Dazhong Wang [a]

[a] Laboratory of Intelligent Control and Robotics, Shanghai University of Engineering Science, Shanghai 201620, China
[b] School of Automation, Image Processing and Intelligent Control Key Laboratory of Education, Ministry of China, Huazhong University of Science and Technology, Wuhan 430074, China
[c] College of Information Science and Technology, Donghua University, Shanghai 201620, China

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ABSTRACT

This paper is considered with the $\mathcal{H}_\infty$ observer design problem for a class of nonlinear systems with the one-sided Lipschitz condition. The systems under consideration include the well-studied Lipschitz system as a special case and possess inherent advantages with respect to conservativeness. For such systems in the presence of noises, we develop a Linear Matrix Inequality (LMI) based approach to design a nonlinear $\mathcal{H}_\infty$ observer by carefully dealing with the one-sided Lipschitz condition together with the quadratic inner-bounded condition. The resulting nonlinear $\mathcal{H}_\infty$ observer guarantees asymptotic stability of the estimation error dynamics with a prescribed $\mathcal{H}_\infty$ performance. Moreover, for the design purpose, the existence condition of the proposed nonlinear $\mathcal{H}_\infty$ observer is formulated in terms of LMIs by using a matrix generalized inverse technique. Finally, a simulation example is given to illustrate the effectiveness of the proposed design.

1. Introduction

The state estimation problem for nonlinear systems has received considerable attention in the past two decades (see, e.g., [1–4] and the references therein). This is partly due to the fact that a state observer is usually required for implementation of the control design when not all the internal states of the system are available. In addition, observers are often employed in many engineering applications, such as process monitoring, fault detection and isolation, neural network [5,6], and chaos synchronization-based secure communication [7,8]. Generally speaking, it is very difficult or even impossible to design a state observer for a general nonlinear system. Therefore, up to now, most of the existing observer design methods are focused on some special forms of nonlinearities. A popular class of nonlinear systems considered by the researchers is the Lipschitz system. As we know, most of the physical system models satisfy the Lipschitz condition, at least locally. In the existing literature, the so-called *Lipschitz nonlinear observer* has been extensively studied (see, e.g., [7–11]).

The conventional Lipschitz condition is commonly used in the existing nonlinear observer design. However, a major limitation in the existing results for Lipschitz nonlinear systems is that most of them work only for adequately small values of the Lipschitz constant [12]. In order to overcome this limitation, the so-called *one-sided Lipschitz condition* was first introduced by Hu [13] for nonlinear state estimation. Several interesting works on state observers and stabilization for this type of systems were recently developed in [14–17]. More recently, Abbaszadeh and Marquez [12] extended the concept of one-sided Lipschitz and proposed a novel approach to design a full-order observer for one-sided Lipschitz nonlinear systems. Based on Riccati equations or the LMI technique, further results on the full-order and reduced-order observers for such systems were developed by Zhang et al. [18,19]. The observer design for discrete-time one-sided Lipschitz systems was addressed in Zhang et al. [20] and Benallouch et al. [21]. It is worth mentioning that the one-sided Lipschitz covers its well-known Lipschitz counterpart as a special case and has inherent advantages with respect to conservativeness [12].

On the other hand, recent literature has witnessed an increasing interest in the study of the state estimation problem for nonlinear systems in the presence of noises (see, e.g., [11,22–30] and the references therein). There are generally two broad approaches to deal with this problem [28]. One is based on the celebrated Kalman filtering and the other is the $\mathcal{H}_\infty$ approach. In the Kalman filtering based approach, the measurement noises are assumed to be Gaussian with known statistics. When the noises are arbitrary signals with bounded energy, $\mathcal{H}_\infty$ filtering provides a guaranteed noise attenuation level [22]. In recent literature, many useful $\mathcal{H}_\infty$ filtering approaches have been developed for several

* Corresponding author.
E-mail address: houshengsu@gmail.com (H. Su).

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kinds of systems [22–30]. For instance, Xu et al. [25] and Wu et al. [26] considered the $\mathcal{H}_\infty$ control and filtering for uncertain Markovian jump systems with time-varying delays. Pertew et al. in [11,24] proposed a new dynamic framework to design the $\mathcal{H}_\infty$ for Lipschitz systems. Abbaszadeh and Marquez et al. [27] and Darouach et al. [28] dealt with the $\mathcal{H}_\infty$ observer design problem for singular Lipschitz nonlinear systems. However, to the best of our knowledge, few results have been given on the study of $\mathcal{H}_\infty$ observers for one-sided Lipschitz nonlinear systems. This motivates our present research.

In this paper, we deal with the nonlinear $\mathcal{H}_\infty$ filtering problem for one-sided Lipschitz systems in the presence of noises. A similar problem has been studied in Tai and Wang [15], where the system is assumed to satisfy the one-sided Lipschitz condition given by Hu [13]. However, how to check this type of one-sided Lipschitz is still an open problem [14]. Therefore, the observer existence conditions given in [15] are not easy to verify. In this work, we use the extended concept of one-sided Lipschitz proposed by Abbaszadeh and Marquez [12]. Firstly, by carefully considering the one-sided Lipschitz together with the quadratic inner-bounded condition, we develop a novel LMI-based approach to design the $\mathcal{H}_\infty$ observer for such a system. A sufficient condition that guarantees both the asymptotic stability and a prescribed $\mathcal{H}_\infty$ performance of the filtering error dynamics is then obtained. Moreover, for the design purpose, we transform the condition into a tractable LMI format through using a matrix generalized inverse technique. We also present an algorithm to design a desired nonlinear $\mathcal{H}_\infty$ observer for the system. Finally, simulation results are provided to illustrate the effectiveness of the proposed design.

Notations: $\mathcal{R}^n$ denotes the n-dimensional real Euclidean space. $\mathcal{R}^{m\times n}$ represents the set of all $m\times n$ real matrices. $\langle , \rangle$ stands for the inner product, i.e., given $x, y \in \mathcal{R}^n$, then $\langle x, y\rangle = x^T y$, where x^T is the transpose of the vector x. $\|\cdot\|$ denotes the Euclidean norm. The notation $\mathcal{L}_2[0,\infty)$ represents the space of square-integrable vector functions over $[0,\infty)$ and $\|\cdot\|_2$ stands for the usual $\mathcal{L}_2$ norm. For a square matrix P, $P>0$ (<0) means that the matrix is symmetric and positive definite (negative definite). In symmetric block matrices, an asterisk '$*$' represents a term that is induced by symmetry. I denotes an identity matrix with appropriate dimension.

2. System description and preliminaries

In this paper, we consider the following nonlinear system described by

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Df(F_L x, u) + D_1\omega(t) \\ y(t) = Cx(t) + D_2\omega(t) \end{cases} \tag{1}$$

where $x(t)\in\mathcal{R}^n$ is the state vector, $u\in\mathcal{R}^m$ is the known input, and $y\in\mathcal{R}^p$ is the measurement output. $\omega(t)\in\mathcal{R}^s$ is the disturbance vector, which contains both system and measurement noises. $A\in\mathcal{R}^{n\times n}$, $B\in\mathcal{R}^{n\times m}$, $C\in\mathcal{R}^{p\times n}$, $D\in\mathcal{R}^{n\times r}$, $F_L\in\mathcal{R}^{r\times n}$, $D_1\in\mathcal{R}^{n\times s}$ and $D_2\in\mathcal{R}^{p\times s}$ are known constant matrices. Without loss of generality, assume that C is of full row rank. The vector function $f(F_L x, u): \mathcal{R}^r\times\mathcal{R}^m\to\mathcal{R}^r$ represents the nonlinear part of the system. Throughout the paper, we assume that $f(F_L x, u)$ satisfies the following two assumptions.

Assumption 1 (*Abbaszadeh and Marquez [12]*). $f(F_L x, u)$ verifies the *one-sided Lipschitz condition*, i.e.,

$$\langle f(F_L x, u) - f(F_L\hat{x}, u), F_L(x-\hat{x})\rangle \le \rho\|F_L(x-\hat{x})\|^2, \tag{2}$$

where $\rho\in\mathcal{R}$ is the so-called *one-sided Lipschitz constant*.

Assumption 2 (*Abbaszadeh and Marquez [12]*). $f(F_L x, u)$ verifies the *quadratic inner-bounded condition*, i.e.,

$$\|f(F_L x, u) - f(F_L\hat{x}, u)\|^2 \le \beta\|F_L(x-\hat{x})\|^2 + \gamma\langle f(F_L x, u) - f(F_L\hat{x}, u), F_L(x-\hat{x})\rangle, \tag{3}$$

where $\beta\in\mathcal{R}$ and $\gamma\in\mathcal{R}$ are known constants.

It should be noted that here the constants ρ, β and γ can be positive, negative or zero, while the well-known Lipschitz constant must be positive. Additionally, if the vector function $f(F_L\hat{x}, u)$ satisfies the Lipschitz condition, then it is also both one-sided Lipschitz and quadratically inner-bounded, but the converse is not true (see [12]). The concept of quadratic inner-boundedness (3) given in [12] is very useful to derive tractable LMI stability conditions of observers.

For system (1), we consider the following state observer:

$$\begin{cases} \dot{\xi}(t) = N\xi(t) + My(t) + TBu(t) + TDf(F_L\hat{x}, u) \\ \hat{x}(t) = G\xi(t) + Fy(t) \end{cases} \tag{4}$$

where $\xi(t)\in\mathcal{R}^q$ represents the state vector of the observer and $\hat{x}(t)\in\mathcal{R}^n$ denotes the estimate of $x(t)$. N, M, T, G, and F are unknown real matrices with appropriate dimensions and will be determined later. Let the error between $\xi(t)$ and $Tx(t)$ be

$$\varepsilon(t) = Tx(t) - \xi(t). \tag{5}$$

Then we can obtain the following dynamics of $\varepsilon(t)$:

$$\dot{\varepsilon}(t) = N\varepsilon + (TA - NT - MC)x + TD\Delta f + \Phi\omega, \tag{6}$$

where

$$\Delta f = f(F_L x, u) - f(F_L\hat{x}, u), \tag{7}$$

$$\Phi = TD_1 - MD_2. \tag{8}$$

And then

$$\hat{x}(t) = -G\varepsilon + (GT + FC)x + FD_2\omega. \tag{9}$$

If the matrix T can be chosen such that

$$NT + MC = TA, \tag{10}$$

$$GT + FC = I_n. \tag{11}$$

Then Eqs. (6) and (9) become

$$\dot{\varepsilon}(t) = N\varepsilon(t) + TD\Delta f + \Phi\omega, \tag{12}$$

$$e(t) = G\varepsilon(t) - FD_2\omega, \tag{13}$$

where $e(t)$ is the estimation error defined by $e(t) = x(t) - \hat{x}(t)$.

With the above analysis, the *nonlinear $\mathcal{H}_\infty$ observer design problem* of system (1) can then be formulated as follows: given a prescribed level of noise attenuation $\mu>0$, find a suitable observer in the form (4), such that the observer estimation error $e(t)$ is asymptotically stable for $\omega(t)=0$ and $\|e(t)\|_2 < \mu\|\omega(t)\|_2$ under zero initial condition for any non-zero $\omega(t)\in\mathcal{L}_2[0,\infty)$.

3. Main results

In this section, we attempt to solve the nonlinear $\mathcal{H}_\infty$ observer design problem of system (1) by developing an LMI-based approach. Notice that for $\omega(t)=0$ we have $e(t)=G\varepsilon(t)$. Therefore, the asymptotic stability of $\varepsilon(t)$ implies that $e(t)\to 0$ as $t\to\infty$. The following theorem gives a sufficient condition for the stability of $e(t)$.

Theorem 1. *Under Assumptions 1 and 2, for $\omega(t)=0$, the observer estimation error $e(t)$ is asymptotically stable, if there exist matrices $P>0$, N, M, T, G, and F and scalars $\tau_1>0$, $\tau_2>0$ such that (10) and (11) are satisfied and the following matrix inequality is*