Neurocomputing 133 (2014) 369–376

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Letters

Probability-dependent H_∞ synchronization control for dynamical networks with randomly varying nonlinearities ☆

Licheng Wang [a], Guoliang Wei [a,*], Wangyan Li [b]

[a] Shanghai Key Lab of Modern Optical System, Department of Control Science and Engineering, University of Shanghai for Science and Technology, Shanghai 200093, China
[b] Business School, University of Shanghai for Science and Technology, Shanghai 200093, China

ARTICLE INFO

Article history:
Received 28 October 2013
Received in revised form
6 December 2013
Accepted 8 December 2013
Communicated by Z. Wang
Available online 10 January 2014

Keywords:
Dynamical networks
Gain-scheduled
Time-varying nonlinearities
Probability-dependent Lyapunov function
Synchronization control

ABSTRACT

In this paper, the H_∞ synchronization control problem is investigated for a class of dynamical networks with randomly varying nonlinearities. The time varying nonlinearities of each node are modelled to be randomly switched between two different nonlinear functions by utilizing a Bernoulli distributed variable sequence specified by a randomly varying conditional probability distribution. A probability-dependent gain scheduling method is adopted to handle the time varying characteristic of the switching probability. Attention is focused on the design of a sequence of gain-scheduled controllers such that the controlled networks are exponentially mean-square stable and the H_∞ synchronization performance is achieved in the simultaneous presence of randomly varying nonlinearities and external energy bounded disturbances. Except for constant gains, the desired controllers are also composed of time varying parameters, i.e., the time varying switching probability and therefore less conservatism will be resulted comparing with traditional controllers. In virtue of semi-definite programming method, controller parameters are derived in terms of the solutions to a series of linear matrix inequalities (LMIs) that can be easily solved by the Matlab toolboxes. Finally, a simulation example is exploited to illustrate the effectiveness of the proposed control strategy.

1. Introduction

Complex networks are consisted of mass numbers of highly interrelated nodes and can be used to model various of practical systems in the real world. In many practical situations, it is very common that a large number of complicated systems have been simplified by complex networks for theoretical analysis such as neural systems, social networks, food webs, power grids and so on. From the end of last century, complex networks have received particularly research attention and an increasing interest since the two fundamental academic papers have been published with the discoveries of the "small-world" and "scale-free" properties [2,27]. Furthermore, very recently, due to the arrival of the age of big data, complex networks are becoming a hot research area in the simultaneous presence of opportunities and challenges and, how to handle the large scale of network data trends to be a crucial technical problem to be solved. On the other hand, synchronization as a complex and interesting phenomenon in the dynamical networks has attracted continual research attentions in the past few years not only because of it being a universal behavior in the natural world and commonly existing in many system models such as, the large-scale and complex networks of chaotic oscillators [13,15–17,19], the coupled systems exhibiting spatiotemporal chaos and autowaves [23,32], and the array of coupled neural networks [8,31], but also owing to its bright prospects in the practical applications in a wide range of fields including parallel image processing [10], pattern storage and retrieval [14], and secure communications.

It is well known that nonlinearities are ubiquitous and inevitable in almost all practical engineering applications and are of probable source for performance degradation, which has posed a great challenge for system design, and many research efforts have been devoted to this main stream topic in the control community during the past several decades, see e.g. [4–7,11,12,24,25]. Due mainly to its own complexity, the dynamical networks easily trend to be subject to a large class of nonlinearities which can be resulted from the additive nonlinear exogenous disturbances caused by environmental circumstances. On the other hand, in the complex dynamical networks, it hardly obtains an explicit description of the nonlinear disturbances in the form of either intensity or types, namely the nonlinear disturbances themselves may suffer from random abrupt changes owing primarily to some

☆This work was supported in part by the National Natural Science Foundation of China under Grant 61074016, the Shanghai Natural Science Foundation of China under Grant 10ZR1421200, the Leverhulme Trust of the UK and the Alexander von Humboldt Foundation of Germany.

* Corresponding author.
E-mail address: guoliang.wei1973@gmail.com (G. Wei).

0925-2312/$ - see front matter © 2014 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.neucom.2013.12.045

abrupt phenomena, such as random failures and repairs of the components, intermittently switching in the interconnections of nodes, sudden environment changes, and modification of the operating point of a linearized model of a nonlinear systems [25]. Hence, such nonlinear disturbances vary in a random way in terms of intensity or types with a determined or uncertain probability distribution, which gives rise to the so-called randomly varying nonlinearities (RVNs). Since the RVNs can reflect the nonlinear disturbances existing in many practical physical processes more properly and accurately, some initial attentions have been focused on it, see e.g. [4,18].

Because of the unpredictable disturbances or the inherent structure of components, there exists a nonzero probability for system parameters to deviate the normal value, and such system is named as time varying system. Rather than time invariant system, the time varying system is another class of important system and more suitable for approximating the practical situations. However, with the parameter perturbation entering into the original system, the difficulties for controller/filter design have been potentially increased, hence, it is very urgent to develop a proper method to overcome this challenge. Fortunately, the gain-scheduling approach has been seen as an effective way for the time varying system, whose main idea is to associate with the scheduling parameters when design controller/filter gains. Owing to its scheduling nature according to the time varying parameters, the gain-scheduling method has indeed reduced the possible conservatism compared with the traditional ones. Parallel to the gain-scheduling technique, parameter-dependent Lyapunov function method is another useful tool to cope with the uncertain system parameters in order to get a desired performance as well as reduce the redundant design introduced by the time-varying parameters and, lots of research results have been appeared in the literature, see e.g. [1,3,26]. Very recently, some reported results have been concerned with the combination of the two advanced techniques rationally for achieving better performance requirements, see e.g. [28,29]. However, up to now, to the best of authors′ knowledge, the synchronization control problem has not been focused on for complex dynamical networks with time-varying nonlinearities via a probability dependent gain-scheduled approach due mainly to the mathematical complexity, which gives us the motivation for further investigation.

Summing up the above discussions, in this paper, we aim to deal with the problem of synchronization control of complex dynamical networks which are subject to randomly varying nonlinearities via a probability dependent gain scheduling approach. A Bernoulli distributed sequence is introduced to account for the stochastic phenomenon of time varying nonlinearities with a dynamical probability that is measurable in real time. The main contribution of this paper is highlighted as follows: (1) it is proposed that a new dynamical networks model covers the randomly varying nonlinearities whose occurrence probability is described by a series of varying Bernoulli distributions yet taking value on a certain interval, which is more close to the practical engineering; (2) the potential conservatism will be reduced resulting from time varying probability distributions via introducing the parameter dependent Lyapunov function and slack variable; (3) an array of dynamical controller gains for complex networks has been developed, which are scheduled with the changeable probability distributions.

The remainder of this technique note is organized as follows: In Section 2, a stochastic dynamical networks model is formulated in the presence of randomly varying nonlinearities and exogenous bounded disturbances. In Section 3, a sufficient condition is provided to guarantee the exponentially stable of dynamical networks and, further more, the controller gain of each node is derived in terms of the solutions to a sequence of linear matrix inequalities (LMIs). An illustrated numerical simulation is given to show the effectiveness and applicability of our proposed algorithm in Section 4 and a conclusion is summarized in Section 5.

Notation: The notations are quite standard. Throughout this paper, $\mathbb{Z}^+$, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the positive integer space, the n-dimensional Euclidean space and the set of all $n\times m$ real matrices. The superscript "T" denotes matrix transposition and the notation $X\geq Y$ (respectively, $X>Y$) where X and Y are symmetric matrices, means that $X-Y$ is positive semi-definite (respectively, positive definite). For vector z, $z\geq 0$ means that each entry of z is nonnegative. I is the identity matrix. $\mathbb{E}\{x\}$ stands for the expectation of stochastic variable x, and $\|x\|$ describes the Euclidean norm of a vector x. If A is a matrix, denote by $\lambda_{\max}(A)$ (respectively, $\lambda_{\min}(A)$) the largest (respectively, smallest) eigenvalue of A. Matrices, if not explicitly specified, are assumed to have compatible dimensions.

2. Problem formulation

Consider the dynamical networks with N coupled nodes as the following form:

$$\begin{cases} x_i(k+1)=Ax_i(k)+\alpha(k)f(x_i(k),k)+(1-\alpha(k))g(x_i(k),k) \\ \qquad\qquad +\sum_{j=1}^{N} w_{ij}\Gamma x_j(k)+u_i(k)+B_i v(k), \\ z_i(k)=Mx_i(k), \end{cases} \tag{1}$$

where $x_i(k)\in\mathbb{R}^n$, $u_i(k)\in\mathbb{R}^n$ and $z_i(k)\in\mathbb{R}^m$ are the state vector, control input and controlled output of the ith node, respectively. $v(k)\in\mathbb{R}^p$ is the disturbance input belonging to $l_2[0,+\infty)$. $f(\cdot)$ and $g(\cdot)$ are nonlinear vector functions. $\Gamma=\text{diag}(\gamma_1,\gamma_2,\ldots,\gamma_n)$ is the inner coupling matrix between two connected nodes for all $1\leq i,j\leq N$. $W=(w_{ij})_{N\times N}$ is the coupled configuration matrix representing the coupling structure of the dynamical networks. If there is a connection between node i and node j $(i\neq j)$, $w_{ij}>0$, otherwise, $w_{ij}=0$. In this paper, as usual, we assume W to be symmetric matrix and satisfy the condition $\sum_{j=1,\,i\neq j}^{N} w_{ij}=-w_{ii}$ $(i=1,2,\ldots,N)$. A, B_i and M are constant matrices with appropriate dimensions.

The vector-value functions $f(\cdot)$ and $g(\cdot)$: $\mathbb{R}^n\to\mathbb{R}^n$ represent two different nonlinear disturbances which are assumed to be continuous and satisfy the following conditions:

$$\|f(x(k))\|^2\leq\delta\|G_1x(k)\|^2, \tag{2}$$

$$\|g(x(k))\|^2\leq\beta\|G_2x(k)\|^2, \tag{3}$$

where δ, β are known positive scalars, and G_1, G_2 are known constant real matrices of appropriate dimensions.

In this paper, we are interested to steer the dynamical network systems to a desired state $s(k)$, which is described as the solution to the following specified reference model:

$$\begin{cases} s(k+1)=As(k), \\ z(k)=Ms(k), \end{cases} \tag{4}$$

where $z(k)$ is the output of the target state. Denoting $e_i(k)=x_i(k)-s(k)$, $\tilde{z}_i(k)=z_i(k)-z(k)$, respectively, the following systems that govern the synchronization error dynamics can be obtained:

$$\begin{cases} e_i(k+1)=Ae_i(k)+\alpha(k)f(e_i(k)+s(k),k) \\ \qquad\qquad +(1-\alpha(k))g(e_i(k)+s(k),k) \\ \qquad\qquad +\sum_{j=1}^{N} w_{ij}\Gamma e_j(k)+u_i(k)+B_i v(k), \\ \tilde{z}_i(k)=Me_i(k), \end{cases} \tag{5}$$

for all $i=1,2,\ldots N$. Stochastic variable $\alpha(k)$ in (1) is a Bernoulli-distributed sequence that accounts for the randomly varying