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Periodically intermittent control on robust exponential synchronization for switched interval coupled networks

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A B S T R A C T

This paper investigates the problem of robust synchronization for switched coupled interval networks via intermittent control. Based on the theories of the switched systems and coupled interval networks, the mathematical model of the switched coupled networks with uncertain parameters is established, the switched coupled interval networks consist of m modes and the interval networks switch from one mode to another according to a switch rule. By applying the average dwell time approach, structuring multiple Lyapunov–Krasovskii functions, and using Halanay inequality, new synchronization criteria are obtained for switched coupled networks with uncertain parameters. Moreover, as a special case, the obtained results can be used to check robust synchronization for coupled networks with uncertain parameters. Finally, an illustrative example is provided to demonstrate the validity of the theoretical results.

1. Introduction

Recently, coupled neural networks [1–3], as a special case of complex systems, consist of a large set of coupled nodes, which maybe chaotic networks, have received increasingly researchers' attentions since they widely exist in our real life such as the internet, genetic networks, social networks, and food webs. Most of the existing researches mainly concentrate on stability analysis for neural networks or coupled networks with or without delays [4–6]. However, synchronization in coupled networks is one of the most important collective dynamical behaviors and has been widely applied in different areas including secure communication, parallel image processing and pattern storage. In [7], authors investigated the global synchronization of N identical delayed neural networks (DNNs) with constant and delayed coupling, it showed that the chaos synchronization of coupled DNNs was ensured by a suitable design of inner coupled linking matrix and the inner delayed linking matrix. In most of the cases, we need coupled networks reach to synchronization, nevertheless, the network cannot synchronize by itself, many continuous or discontinuous control methods have been developed to ensure the network to synchronize [8–10]. In [8], by using Lyapunov functions and analysis technique, the robust synchronization was considered for uncertain coupled delayed networks under general impulsive control. In [9], authors presented some criteria for nonlinear systems by designing linear feedback controllers. In [10], the synchronization of an uncertain dynamical network by adding an adaptive controller to each node was discussed. But in practical application, it is too costly to add controllers with continuous feedback, to reduce the control time, intermittent control and impulsive control are proposed. It is worth noting that both impulsive control and intermittent control are discontinuous controls, the difference between them is that impulsive control only acts on some isolated points, while intermittent control is activated during some intervals and does not work over the other intervals. Intermittent control, which was first introduced to control nonlinear dynamical systems [11], has been used for a variety of purposes such as manufacturing, transportation and communication. However, there are few results to study synchronization via intermittent control [12–17]. Authors studied exponential stochastic synchronization problem for coupled networks by adding different intermittent controllers [12]. In [13], assumption that the control period and control width were fixed and known, a suboptimal intermittent controller was designed to study the exponential stabilization problem for a class of nonlinear systems. To the best of our knowledge, there are few published papers that deal with the problems of synchronization for switched coupled networks by using intermittent control.

Switched networks [18] consist of a set of individual subsystems and a switch rule, which have attracted significant attentions and witnessed the successful applications in many fields such as computer communities, artificial intelligence, automotive industry, gene selection in a DNA microarray analysis and electric power systems. Therefore, the stability issues of switched neural networks have been investigated [19–22], most of the previous results focus on the stability of switched neural networks under arbitrary switching rule by using

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common Lyapunov function method. However, common Lyapunov function method requires all the subsystems of the switched system [23–26] to share a positive definite radially unbounded common Lyapunov function. Generally, this requirement is difficult to achieve. To avoid the common Lyapunov function and obtain less conservative stability conditions, the average dwell time method is proposed to deal with the analysis and synthesis of switched networks, which is regarded as an important and attractive method to find a suitable switching signal to guarantee switched system stability or improve other performance, and has been widely applied to investigate the analysis and synthesis for switched system with or without time-delay. In [27], based on multiple Lyapunov functions method and LMI techniques, the authors presented some sufficient conditions in terms of LMIs which guaranteed the robust exponential stability for uncertain switched Cohen–Grossberg neural networks with interval time-varying delay and distributed time-varying delay under the switching rule with the average dwell time property. In [28], employing the average dwell time approach (ADT), stochastic analysis technology, some mean-square exponential stability criteria for the switched stochastic system were presented in terms of LMIs.

Most of the existing works focus only on the coupled networks in deterministic forms, in reality, due to unavoidable factors, such as modeling error, external perturbation and parameter fluctuation, the networks model certainly involve uncertainties such as perturbations and component variations, which can greatly affect the dynamical behaviors of networks. To analyze robustness of networks [29,30], one reasonable method is to assume parameters in certain intervals. In spite of these advances in studying robust of coupled networks, robust synchronization for coupled interval networks has not been investigated in the literature. Therefore, it is of great importance to study robust synchronization of switched interval coupled networks.

Motivated by the aforementioned discussions, we deal with the robust synchronization problem for switched coupled networks with uncertain parameters via intermittent control in this paper, by using methods of the average dwell time, utilizing multiple Lyapunov–Krasovskii functional and the Kronecker product, using Halanay inequality, new synchronization criteria are obtained for switched coupled networks with uncertain parameters. The main novelty of this paper can be summarized as following: (1) introduce switch idea of average dwell time to coupled networks, (2) consider the interval parameters fluctuation, a new mathematical model of the switched coupled networks with parameters in interval is established, it becomes much closer to the actual model, and (3) new synchronization criteria are obtained for switched coupled networks with uncertain parameters, which can be used to check robust synchronization for coupled networks with uncertain parameters.

The rest of this paper is organized as follows. In Section 2, the model formulation and some preliminaries are presented. In Section 3, some robust exponential synchronization criteria for switched interval coupled networks are obtained by intermittent control. An numerical example is given to demonstrate the validity of the proposed results in Section 4. Some conclusions are drawn in Section 5.

Notations: Throughout this paper, for any matrix A, $A \geq 0$ $(A \leq 0)$ means that A is semi-positive definite (semi-negative definite). A^T denotes the transpose of A. $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the maximum and minimum eigenvalue of A respectively. The norm of piecewise right continuous function $\nu(t)$ is denoted by $\|\nu\|_\tau = \sup_{-\tau \leq s \leq 0} \|\nu(t+s)\|$. The vector norm is denoted by $\|\bullet\|$, is taken to be Euclidian. Matrices, if their dimensions not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2. Coupled networks model and preliminaries

The model of interval coupled networks consisting of N identical nodes with linearly diffusive couplings can be described by differential equation system:

$$\begin{cases} \dot{x}_i(t) = -Ax_i(t) + B_1 f(x_i(t)) + B_2 g(x_i(t-\tau)) \\ \qquad + \sum_{j=1}^{N} G_{ij}\Gamma x_j(t) + I(t), \\ A \in A_I, \quad B_k \in B_I^{(k)}, \ k=1,2, \end{cases} \tag{1}$$

where $x_i(t) = (x_{i1}(t), \ldots, x_{in}(t))^T \in R^n$ is the state vector of the ith dynamical node; time delay $\tau > 0$; $f(x_i(\cdot))$, $g(x_i(\cdot))$ are the vector-valued neuron activation functions, $f(x_i(t)) = (f_1(x_{i1}(t)), \ldots, f_n(x_{in}(t)))^T \in R^n$, $g(x_i(t)) = (g_1(x_{i1}(t)), \ldots, g_n(x_{in}(t)))^T \in R^n$; $I(t)$ is a real external input vector of each node; $\Gamma \in R^{n\times n}$ is the inner-coupling matrix, $G = (G_{ij})_{N\times N}$ is the coupling configuration matrix representing the topological structure of the system, if there is a link between node i and node j, then $G_{ij} > 0 (j \neq i)$, otherwise, $G_{ij} = 0$. Suppose G is irreducible matrix and satisfies the diffusive coupling condition, that is $\sum_{j=1}^{N} G_{ij} = 0$. $A = \mathrm{diag}(a_1, \ldots, a_n) > 0$ is an $n \times n$ constant diagonal matrices, denotes the rate with which the cell i resets its potential to the resting state when being isolated from other cells and inputs; $B_k = (b_{ij}^{(k)}) \in R^{n\times n}, k=1,2,$ represent the connection weight matrices, and $A_I = [\underline{A}, \overline{A}] = \{A = \mathrm{diag}(a_i) : 0 < \underline{a}_i \leq a_i \leq \overline{a}_i, i=1,2,\ldots,n\}$, $B_I^{(k)} = [\underline{B}_k, \overline{B}_k] = \{B_k = (b_{ij}^{(k)}) : \underline{b}_{ij}^{(k)} \leq b_{ij}^{(k)} \leq \overline{b}_{ij}^{(k)},\ i,j=1,2,\ldots,n\}$ with $\underline{A} = \mathrm{diag}(\underline{a}_1, \underline{a}_2, \ldots, \underline{a}_n)$, $\overline{A} = \mathrm{diag}\ (\overline{a}_1, \overline{a}_2, \ldots, \overline{a}_n)$, $\underline{B}_k = (\underline{b}_{ij}^{(k)})_{n\times n}$, $\overline{B}_k = (\overline{b}_{ij}^{(k)})_{n\times n}$.

Throughout this paper, the following assumptions are made on activation functions f and g:

($\mathcal{H}$): For any two different $x,\ y \in R^n$,

$$\|f(x) - f(y)\| \leq L_f \|(x-y)\|,$$
$$\|g(x) - g(y)\| \leq L_g \|(x-y)\|,$$

where L_f, L_g are positive constants.

Let $s(t)$ be a solution of an isolated node, then

$$\begin{cases} \dot{s}(t) = -As(t) + B_1 f(s(t)) + B_2 g(s(t-\tau)) + I(t), \\ A \in A_I, \quad B_k \in B_I^{(k)}, \ k=1,2, \end{cases} \tag{2}$$

$s(t)$ may be an equilibrium point, a periodic orbit, or even a chaotic attractor.

Based on some transformations [30], the system (1) can be equivalently written as

$$\dot{x}_i(t) = -[A_0 + E_A \Sigma_A F_A]x_i(t) + [B_{10} + E_1 \Sigma_1 F_1] f(x_i(t)) + [B_{20} + E_2 \Sigma_2 F_2] g(x_i(t-\tau)) + \sum_{j=1}^{N} G_{ij}\Gamma x_j(t) + I(t), \tag{3}$$

where $\Sigma_A \in \Sigma,\ \Sigma_k \in \Sigma,\ k=1,2.$

$$\Sigma = \left\{ \mathrm{diag}[\delta_{11}, \ldots, \delta_{1n}, \ldots, \delta_{n1}, \ldots, \delta_{nn}] \in R^{n^2\times n^2} : |\delta_{ij}| \leq 1,\ i,j=1,2,\ldots,n \right\}.$$

$$A_0 = \frac{\overline{A} + \underline{A}}{2}, \quad H_A = [\alpha_{ij}]_{n\times n} = \frac{\overline{A} - \underline{A}}{2} \cdot B_{k0} = \frac{\overline{B}_k + \underline{B}_k}{2},$$

$$HB(k) = [\beta_{ij}]_{n\times n} = \frac{\overline{B}_k - \underline{B}_k}{2}.$$

$$E_A = [\sqrt{\alpha_{11}}e_1, \ldots, \sqrt{\alpha_{1n}}e_1, \ldots, \sqrt{\alpha_{n1}}e_n, \ldots, \sqrt{\alpha_{nn}}e_n]_{n\times n^2},$$
$$F_A = [\sqrt{\alpha_{11}}e_1, \ldots, \sqrt{\alpha_{1n}}e_n, \ldots, \sqrt{\alpha_{n1}}e_1, \ldots, \sqrt{\alpha_{nn}}e_n]^T_{n^2\times n},$$
$$E_k = [\sqrt{\beta_{11}^{(k)}}e_1, \ldots, \sqrt{\beta_{1n}^{(k)}}e_1, \ldots, \sqrt{\beta_{n1}^{(k)}}e_n, \ldots, \sqrt{\beta_{nn}^{(k)}}e_n]_{n\times n^2},$$
$$F_k = [\sqrt{\beta_{11}^{(k)}}e_1, \ldots, \sqrt{\beta_{1n}^{(k)}}e_n, \ldots, \sqrt{\beta_{n1}^{(k)}}e_1, \ldots, \sqrt{\beta_{nn}^{(k)}}e_n]^T_{n^2\times n},$$

where $e_i \in R^n$ denotes the column vector with ith element to be 1 and others to be 0.

System (3) can be changed as

$$\dot{x}_i(t) = -A_0 x_i(t) + B_{10} f(x_i(t)) + B_{20} g(x_i(t-\tau)) + E\Delta(t) + \sum_{j=1}^{N} G_{ij}\Gamma x_j(t) + I(t), \tag{4}$$