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Letters

Distributed containment control of linear multi-agent systems

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ABSTRACT

This paper investigates the distributed containment control problem for linear multi-agent systems. Distributed dynamic output feedback controllers on the basis of the relative outputs of neighboring agents are proposed. Necessary and sufficient containment control conditions are presented which are less conservative than those in the literature. These conditions depend on the spectral properties of the topology matrix. Effective algorithms are proposed to obtain control gain matrices based on H_∞ type Riccati design. Then, distributed static output feedback control method is also discussed. Simulation examples are provided finally to demonstrate the effectiveness of the proposed design methods.

1. Introduction

Recently, cooperative control in networks of autonomous mobile agents is extensively studied due to its extensive applications in biological systems, mobile robots, sensor networks, spacecraft formation flying, and other areas. As one of the most fundamental research topics in the field of cooperative control of multi-agent systems, consensus plays an important role in achieving collective behavior through local interactions of agents and has attracted considerable attention. Consensus means that the states of all agents reach an agreement on a common value via local information exchange. Following many pioneering works [1–4], various consensus problems of multi-agent systems were studied, such as consensus of systems with second-order dynamics [5–8] and high-order dynamics [9–12], consensus of agents with time-delay [13,14], agreement over random networks [15–17], and consensus with H_∞ control [18,19], just to mention a few.

Compared to the leaderless consensus problem, a particularly interesting topic is the consensus of a group of agents with a single leader or multiple leaders, where the motion of the leaders is independent of the other following agents. In the single-leader case, the followers are to be driven to approach the leader. Such a problem is named the leader-following consensus problem, and can also be called the cooperative tracking control problem. Many significant results on leader-following consensus have been reported, see, for instance, [20–23] and the references therein. When the multiple leaders are taken into account, the followers will move in the minimum geometric space spanned by the leaders by utilizing appropriate control protocols, that is called the containment control problem. The containment control problem has many practical applications. For instance, a group of robots move to a target when only a few robots can detect the hazardous obstacle. These robots can be designed as leaders, whereas the others can be designed as followers. The followers must stay in the moving safe area formed by the leaders to reach the target safely [26]. In [24], a hybrid control scheme was proposed and the partial differential equation theory was utilized. In [25], a stop-and-go strategy was provided for a group of single-integrator agents under a fixed undirected network topology. Distributed containment control with stationary or dynamic leaders under directed networks was studied in [26] and [27], where single-integrator dynamics and double-integrator dynamics were, respectively, focused on. In [28], finite-time containment control algorithms for autonomous agents with double-integrator dynamics were proposed by using only position measurements. When the multiple Lagrangian systems and multiple rigid bodies were considered, the containment control results can be found in [29] and [30], respectively.

Note that in some applications, the dynamics of the agents are complicated, and cannot be modeled by single- or double-integrator dynamics. Very recently, the containment control problem for multi-agent systems with general linear dynamics was discussed in [31] and [32]. By classifying the agents into internal agents and boundary agents, the continuous-time case was studied in [31]. Both the continuous-time and discrete-time cases were considered in [32]. However, the proposed containment

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condition in [32] was only sufficient. Furthermore, there is no results on containment control using distributed static output feedback controllers, which are simpler than dynamic output feedback controller.

In the current paper, distributed containment control for linear multi-agent systems based on the relative outputs of neighboring agents is studied. The contributions of the paper are as follows. First, distributed dynamic output feedback controllers with distributed observers are proposed. Second, by using spectral analysis and matrix theory, necessary and sufficient containment conditions are presented which are more refined than those in the literature. Third, distributed static output feedback controllers are designed, where the observer is not needed.

Notations. Throughout this paper, for real symmetric matrices X and Y, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X-Y$ is positive semi-definite (respectively, positive definite). I denotes an identity matrix of appropriate dimension. $\mathbf{1}_N \in \mathbb{R}^N$ be the vector with all entries being 1. $\mathbf{0}_{M\times N} \in \mathbb{R}^{M\times N}$ be the matrix with all entries being 0. The notation '$*$' is used as an ellipsis for terms that are induced by symmetry. The Kronecker product of matrices X and Y is denoted as $X \otimes Y$. X^{-1} denotes the inverse matrix of matrix X. $\Lambda(X)$ denotes the set of the eigenvalues of X and $\Lambda^+(X)$ the set of the eigenvalues with positive real part of X. $\mathcal{R}(\lambda)$ denotes the real part of a complex number λ. The convex hull of a finite set of points $x_1, x_2, \dots, x_n \in \mathbb{R}^m$ is the minimal convex set containing all points $x_i, i=1,2,\dots,n$, denote by $\mathrm{co}\{x_1, x_2, \dots, x_n\} = \{\sum_{i=1}^n \alpha_i x_i | \alpha \in \mathbb{R}, \alpha \geq 0, \sum_{i=1}^n \alpha_i = 1\}$.

2. Preliminaries

In this section, some basic concepts and definitions about graph theory and model formulation are briefly introduced.

Let $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ be a directed graph with the set of nodes $\mathcal{V} = \{1,2,\dots,N\}$, the set of directed edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. A directed edge e_{ij} in network $\mathcal{G}$ is denoted by the ordered pair of nodes (i,j), meaning that node j can receive information from node i. The set of neighbors of node i is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} : (j,i) \in \mathcal{E}, j \neq i\}$. The elements of the adjacency matrix D are defined as $d_{ij} = 1$ if and only if there is a directed edge $(j,\ i)$ in $\mathcal{G}$; otherwise, $d_{ij} = 0$. The Laplacian matrix $L = (l_{ij})_{N\times N}$ is defined as $l_{ii} = \sum_{j=1, j\neq i}^N d_{ij}$, and $l_{ij} = -d_{ij}, i \neq j$.

A directed path is a sequence of nodes $1,2,\dots,r$ such that $(i,i+1) \in \mathcal{E},\ i \in \{1,2,\dots,r-1\}$. A directed graph is strongly connected if there is a directed path for any two distinct nodes j and i. A directed graph has a directed spanning tree if there exists at least one node called root node which has a directed path to all the other nodes.

Consider a group of N identical agents with general linear dynamics:

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t),$$
$$y_i(t) = Cx_i(t), \quad i=1,2,\dots,N, \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$, $u_i(t) \in \mathbb{R}^p$ and $y_i(t) \in \mathbb{R}^q$ are the state, the control input and the measured output of agent i, respectively.

An agent is called a leader if the agent has no neighbor, whereas an agent is called a follower if the agent has at least one neighbor. Without loss of generality, assume that the agents indexed by $1,2,\dots,M$ $(M<N)$ are leaders, whereas the rest agents indexed by $M+1, M+2, \dots, N$ are followers. Denote by $i \in \mathcal{L} = \{1,2,\dots,M\}$ the leader, and $i \in \mathcal{F} = \{M+1, M+2, \dots, N\}$ the follower.

Note that the leaders have no neighbors, L becomes

$$L = \begin{bmatrix} \mathbf{0}_M & \mathbf{0}_{M\times(N-M)} \\ L_1 & L_2 \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ L_{11} & L_{12} & \cdots & L_{1M} & L_2 \end{bmatrix}. \tag{2}$$

Assumption 1. *For each follower, there exists at least one leader that has a directed path to that follower.*

Lemma 1 (*Meng et al. [30]*). *Under Assumption 1, all the eigenvalues of L_2 have positive real parts, each entry of $-L_2^{-1}L_1$ is nonnegative, and each row of $-L_2^{-1}L_1$ has a sum equal to 1.*

Definition 1. The containment control is achieved for the agents in (1) if the states of the followers asymptotically converge to the convex hull formed by the leaders.

3. Cooperative dynamic output feedback controller design

In this section, a cooperative dynamic regulator is proposed that uses only the neighbors' output measurements of each agent. A necessary and sufficient condition for containment control is given.

Consider system (1). Denote $\hat{x}_i(t) \in \mathbb{R}^N$ as the estimate of the state $x_i(t)$, $\hat{y}_i(t) = C\hat{x}_i(t)$ as the estimate of output $y_i(t)$. Let $\tilde{x}_i(t) = x_i(t) - \hat{x}_i(t)$ and $\tilde{y}_i(t) = y_i(t) - \hat{y}_i(t)$ as the state estimation error and output estimation error for agent i, respectively.

Propose the following containment control protocol:

$$u_i(t) = K\sum_{j\in\mathcal{N}_i} d_{ij}(\hat{x}_i(t) - \hat{x}_j(t)),$$
$$\dot{\hat{x}}_i(t) = A\hat{x}_i(t) + Bu_i(t) + F\sum_{j\in\mathcal{N}_i} d_{ij}[C(\hat{x}_i(t) - \hat{x}_j(t))], \tag{3}$$

where feedback gain K and observer gain F are to be designed. It is evident that the protocol (3) is distributed, and $u_i(t) = 0, i \in \mathcal{L}$.

Then, we have

$$\dot{x}_i(t) = Ax_i(t) + BK\sum_{j\in\mathcal{N}_i} d_{ij}(x_i(t) - x_j(t)) - BK\sum_{j\in\mathcal{N}_i} d_{ij}(\tilde{x}_i(t) - \tilde{x}_j(t)),$$
$$\dot{\tilde{x}}_i(t) = A\tilde{x}_i + F\sum_{j\in\mathcal{N}_i} d_{ij}[C(\tilde{x}_i(t) - \tilde{x}_j(t))]. \tag{4}$$

Denote

$$\mathcal{X}_i = [x_i^T, \tilde{x}_i^T]^T,$$
$$\mathcal{X} = [\mathcal{X}_1^T, \dots, \mathcal{X}_M^T, \dots \mathcal{X}_N^T]^T = [\mathcal{X}_l^T, \mathcal{X}_f^T]^T,$$
$$\tilde{x} = [\tilde{x}_1^T, \dots, \tilde{x}_M^T, \dots \tilde{x}_N^T]^T = [\tilde{x}_l^T, \tilde{x}_f^T]^T.$$

One has

$$\dot{\mathcal{X}}(t) = (I_N \otimes R + L \otimes S)\mathcal{X}(t), \tag{5}$$

where

$$R = \begin{bmatrix} A & \mathbf{0} \\ \mathbf{0} & A \end{bmatrix}, \quad S = \begin{bmatrix} BK & -BK \\ \mathbf{0} & FC \end{bmatrix}.$$

The next result gives a necessary and sufficient condition for the containment control of system (1) with protocol (3).

Theorem 1. *Suppose that (A, B) is stabilizable, (A, C) is detectable, and Assumption 1 holds. Then, containment control of system (1) can be achieved under protocol (3) if and only if all matrices $A+\lambda BK$, $A+\lambda FC$, $\lambda \in \Lambda^+(L)$ are Hurwitz. In particular, $x_f(t) \to -L_2^{-1}L_1 \otimes e^{At}x_l(0)$, $\tilde{x}_f(t) \to -L_2^{-1}L_1 \otimes e^{At}\tilde{x}_l(0)$ as $t \to \infty$.*

Proof (*Sufficiency*). From the property of L and Assumption 1, we know that the algebraic multiplicity of eigenvalue 0 of L is M. It can be verified that the geometric multiplicity of eigenvalue 0 of L is M. Therefore, L can be expressed as the following form:

$$L = PJP^{-1}$$