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Robust state estimation for discrete-time BAM neural networks with time-varying delay

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ABSTRACT

This paper is concerned with the robust delay-dependent state estimation problem for a class of discrete-time Bidirectional Associative Memory (BAM) neural networks with time-varying delays. By using the Lyapunov–Krasovskii functional together with linear matrix inequality (LMI) approach, a new set of sufficient conditions are derived for the existence of state estimator such that the error state system is asymptotically stable. More precisely, an LMI-based state estimator and delay-dependent stability criterion for delayed BAM neural networks are developed. The conditions are established in terms of LMIs which can be solved by the MATLAB LMI toolbox. It should be mentioned that all the sufficient conditions are dependent on the upper and lower bounds of the delays. Also, the desired estimator unknown gain matrix is determined in terms of the solution to these LMIs. Finally, numerical examples with simulation results are given to illustrate the effectiveness and applicability of the obtained results.

1. Introduction

In the past few years, various classes of neural networks have been developed and studied, due to their practical importance and successful applications in many areas such as optimization, pattern recognition, signal processing, image processing, fault diagnosis, associative memory and so on [1,2]. However, such applications heavily depend on the study of dynamical analysis of neural networks in the presence of time delays and parametric uncertainties [3–5]. In particular, time delays may cause undesirable dynamic network behaviors such as oscillation and instability and the connection weights of neurons depend on certain resistance and capacitance values that include uncertainties, such as modeling errors. Kosko [6] proposed a new class of networks called bidirectional associative memory neural networks which is composed of neurons arranged in two layers and the neurons in one layer are fully interconnected to the neurons in the other layer, while there are no interconnect among neurons in the same layer. In recent years, stability analysis problem for BAM neural networks with delays has attracted and interesting results have been reported in [7–9].

On the other hand, in relatively large-scale neural networks, normally only partial information about the neuron states is available in the network outputs [10]. Therefore, in order to make use of the neural networks in practice, it is important and necessary to estimate the neuron states through available measurements [11]. Balasubramaniam et al. [12] studied delay-dependent robust exponential state estimation problem for Markovian jumping fuzzy neural networks with discrete and distributed delays. A delay-independent condition was derived for a class of recurrent neural networks to guarantee the existence of the state estimator by using the delay partition approach in [13]. The state estimation problem for a class of neural networks with time-varying delay is studied in [14,15] and in which sufficient conditions for the existence of estimators have been obtained in terms of LMIs. An improved delay-dependent stability criterion is derived in terms of linear matrix inequalities for the state estimation of fuzzy cellular neural networks with time delay in the leakage term, discrete and unbounded distributed delays [16].

Further, it is noted that neural network problems are extensively studied with continuous-time cases. The discrete-time neural networks become more important than the continuous-time counterparts when implementing the neural networks in a digital way [17]. Recently, the state estimation problem for discrete-time neural networks has received much attention [18–20]. Bao and Cao [21] established a delay distribution dependent condition for the state estimation problem of a class of discrete-time stochastic neural networks with random delays by

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employing Lyapunov technique. The problem of state estimation for a class of discrete-time coupled uncertain stochastic complex networks with missing measurements and time-varying delay by employing the Lyapunov functional method combined with the stochastic analysis approach has been studied in [22] and in which several delay-dependent conditions are established in terms of the solution to certain linear matrix inequalities that ensure the existence of the desired estimator gains. The state estimation problem for discrete-time neural networks with time-varying delay has been studied in [23], where a delay dependent sufficient condition has been presented such that the estimation error system is globally exponentially stable. In [24], the problem of stochastic state estimation has been studied for neural networks with distributed delays and Markovian jump, and a delay-dependent condition proposed to guarantee the exponential stability of the error-state system. More recently, Bao and Cao [25] discussed the problem of robust state estimation for uncertain stochastic bidirectional associative memory networks with time-varying delays, in which a set of sufficient conditions are established in terms of linear matrix inequalities to guarantee the estimation error dynamics to be globally robustly asymptotically stable in the mean square.

However, it should be pointed out to the best of our knowledge, no work has been reported on the state estimation of BAM neural networks with time-varying delay in the discrete case. Motivated by this consideration, the aim of this paper is to derive sufficient condition for state estimation of discrete-time BAM neural networks with time-varying delays. Based on a proper Lyapunov functional, a delay dependent condition is developed for the existence of the desired state estimator via the LMI approach. Further, the result is extended to develop a proper state estimator to approximate the neuron states of uncertain BAM neural networks with time-varying delays, the so-called robust state estimation through available output measurements such that the dynamics of the error-state system is robustly asymptotically stable. The parameter uncertainties are assumed to be norm bounded. The derived conditions are established in terms of LMIs which can be calculated by MATLAB LMI toolbox. Finally, numerical examples with simulation results are provided to illustrate the applicability of the developed results.

2. Problem formulation and preliminaries

In this section, we start by introducing some notations and basic results that will be used in this paper. The superscripts T and (-1) stand for matrix transposition and matrix inverse respectively; $\mathbb{R}^{n\times n}$ denotes the $n\times n$-dimensional Euclidean space; the notation $P>0$ means that P is real, symmetric and positive definite; I and 0 denote the identity and zero matrix with compatible dimensions; diag$\{\cdot\}$ denotes the block-diagonal matrix; we use an asterisk $(*)$ to represent a term that is induced by symmetry. Matrices which are not explicitly stated are assumed to be compatible for matrix multiplications.

Consider the discrete-time BAM neural networks with time-varying delay in the following form:

$$x(t+1)=A_x x(t)+B_x f(y(t))+C_x f(y(t-d(t))),$$
$$y(t+1)=A_y y(t)+B_y g(x(t))+C_y g(x(t-h(t))), \tag{1}$$

where $x(t)=[x_1(t),x_2(t),\ldots,x_n(t)]^T\in\mathbb{R}^n$ and $y(t)=[y_1(t),y_2(t),\ldots,y_m(t)]^T\in\mathbb{R}^m$ denote the neuron state vectors; $A_x=\mathrm{diag}\{a_{1x},a_{2x},\ldots,a_{nx}\}$ and $A_y=\mathrm{diag}\{a_{1y},a_{2y},\ldots,a_{my}\}$ are the state feedback co-efficient matrices with entries $|a_{ix}|<1$, $|a_{jy}|<1$, $i=1,2\ldots n, j=1,2\ldots m$; B_x, B_y are the connection weights and C_x, C_y are the delayed connection weights; $f(y(t))=[f_1(y_1(t)),f_2(y_2(t)),\ldots,f_n(y_n(t))]^T$, and $g(x(t))=[g_1(x_1(t)),g_2(x_2(t)),\ldots,g_m(x_m(t))]^T$ denote the neuron activation functions; the positive integers $h(t)$ and $d(t)$ denote the time-varying discrete delays satisfying

$$h_m\le h(t)\le h_M,\quad d_m\le d(t)\le d_M, \tag{2}$$

where h_m, h_M, d_m and d_M are positive constants. When parameter uncertainty enters into the BAM neural networks (1), the model of discrete-time BAM neural networks with time-varying delays can be formulated as follows:

$$x(t+1)=A_x(t)x(t)+B_x(t)f(y(t))+C_x(t)f(y(t-d(t))),$$
$$y(t+1)=A_y(t)y(t)+B_y(t)g(x(t))+C_y(t)g(x(t-h(t))), \tag{3}$$

where $A_x(t)=A_x+\Delta A_x(t)$, $B_x(t)=B_x+\Delta B_x(t)$, $C_x(t)=C_x+\Delta C_x(t)$, $A_y(t)=A_y+\Delta A_y(t)$, $B_y(t)=B_y+\Delta B_y(t)$ and $C_y(t)=C_y+\Delta C_y(t)$.

Further, the parameter uncertainties $\Delta A_x(t)$, $\Delta B_x(t)$, $\Delta C_x(t)$, $\Delta D_x(t)$, $\Delta A_y(t)$, $\Delta B_y(t)$, $\Delta C_y(t)$ and $\Delta D_y(t)$ are described by

$$[\Delta A_x(t)\ \ \Delta B_x(t)\ \ \Delta C_x(t)\ \ \Delta D_x(t)]=M_xF_x(t)[N_{1x}\ \ N_{2x}\ \ N_{3x}\ \ N_{4x}],$$
$$[\Delta A_y(t)\ \ \Delta B_y(t)\ \ \Delta C_y(t)\ \ \Delta D_y(t)]=M_yF_y(t)[N_{1y}\ \ N_{2y}\ \ N_{3y}\ \ N_{4y}], \tag{4}$$

where N_{1x}, N_{2x}, N_{3x}, N_{4x}, N_{1y}, N_{2y}, N_{3y}, N_{4y}, M_x and M_y are known constant matrices of appropriate dimensions and $F_x(t)$, $F_y(t)$ are unknown time-varying matrices with Lebesgue measurable elements bounded by, $F_x^T(t)F_x(t)\le I$ and $F_y^T(t)F_y(t)\le I$.

Moreover, the activation functions satisfy the following assumptions:

(**H1**) For any $i=1,2,\ldots,n$, there exist constants F_{1i}^-, F_{1i}^+, F_{2i}^- and F_{2i}^+ such that

$$F_{1i}^-\le\frac{g_i(x_1)-g_i(x_2)}{x_1-x_2}\le F_{1i}^+\quad \text{for all } x_1,x_2\in\mathbb{R},\ \ x_1\ne x_2,$$
$$F_{2i}^-\le\frac{f_i(y_1)-f_i(y_2)}{y_1-y_2}\le F_{2i}^+\quad \text{for all } y_1,\ y_2\in\mathbb{R},\ \ y_1\ne y_2.$$

For presentation convenience, in the following we denote

$$C_{11}=\mathrm{diag}\{F_{11}^-F_{11}^+,F_{12}^-F_{12}^+,\ldots,F_{1n}^-F_{1n}^+\},$$
$$C_{12}=\mathrm{diag}\left\{\frac{F_{11}^-+F_{11}^+}{2},\frac{F_{12}^-+F_{12}^+}{2},\ldots,\frac{F_{1n}^-+F_{1n}^+}{2}\right\},$$
$$C_{21}=\mathrm{diag}\{F_{21}^-F_{21}^+,F_{22}^-F_{22}^+,\ldots,F_{2n}^-F_{2n}^+\},$$
$$C_{22}=\mathrm{diag}\left\{\frac{F_{21}^-+F_{21}^+}{2},\frac{F_{22}^-+F_{22}^+}{2},\ldots,\frac{F_{2n}^-+F_{2n}^+}{2}\right\}.$$

The neuron states in relatively large scale neural networks are not often completely available in the network outputs. Therefore, one often needs to estimate the neuron states through available measurements and then utilizes the estimated neuron states to achieve certain design objectives [25]. We consider the following network measurement outputs:

$$z_1(t)=(D_x+\Delta D_x(t))x(t),$$
$$z_2(t)=(D_y+\Delta D_y(t))y(t), \tag{5}$$

where D_x and D_y are the known constant matrix with appropriate dimensions.

The main objective of this paper is to estimate the state of BAM neural networks (1) from the network outputs given in (5), for this we assume the state estimator in the following form:

$$\hat{x}(t+1)=A_x\hat{x}(t)+B_xf(\hat{y}(t))+C_xf(\hat{y}(t-d(t)))+K_1(z_1(t)-D_x\hat{x}(t)),$$
$$\hat{y}(t+1)=A_y\hat{y}(t)+B_yg(\hat{x}(t))+C_yg(\hat{x}(t-h(t)))+K_2(z_2(t)-D_y\hat{y}(t)), \tag{6}$$

where $\hat{x}(t)\in\mathbb{R}^n$, $\hat{y}(t)\in\mathbb{R}^m$ are the estimations of $x(t)$, $y(t)$ respectively and $K_1\in\mathbb{R}^{n\times k}$, $K_2\in\mathbb{R}^{m\times k}$ are the estimator gain matrices to be designed.