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Existence and global exponential stability of anti-periodic solutions for competitive neural networks with delays in the leakage terms on time scales ☆

Yang Liu, Yongqing Yang *, Tian Liang, Li Li

Key Laboratory of Advanced Process Control for Light Industry (Ministry of Education), School of Science, Jiangnan University, Wuxi 214122, PR China

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ABSTRACT

In this paper, the existence and the global exponential stability of anti-periodic solution for competitive neural networks with delays in the leakage terms are investigated on time scales which unifies the continuous-time and the discrete-time competitive neural networks under the same framework. Firstly, the existence of anti-periodic solution is discussed by using the method of coincidence degree and M-matrices. Then some sufficient conditions are obtained to guarantee the global exponential stability of anti-periodic solution for such neural networks. The obtained results are new and improve some earlier publications. Finally, two examples are given to illustrate the effectiveness of the theoretical results.
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1. Introduction

During the last few decades, there had emerged a considerable interest in the study of neural network dynamics. Especially, the competitive neural networks (CNNs) as one of the popular artificial neural networks were investigated by many researchers [4–15]. In this model, there are two types of state variables, the short-term memory (STM) and the long-term memory (LTM). The STM describes rapid changes in neuronal dynamics, and the LTM describes the slow behavior of the unsupervised neural cell synaptic. The detailed hardware implementation for CNNs can be found in [1].

According to related research, a competitive neural network is a kind of unsupervised learning network. It simulates biological systems, which depend on the excitement, coordination, inhibition and competition to process information between neurons, and is widely applied in the image processing, pattern recognition, signal processing, optimization and control theory and so on [1–4].

In 1983, Cohen and Grossberg [4] proposed CNNs. Then many researchers paid attention to the stability analysis of CNNs [5–15]. Meyer studied the various stabilities of CNNs without time delays [5–10]. Based on the theory of flow invariance and Lyapunov functions, the global exponential stability of CNNs was discussed [6] and local exponential stability was investigated [7]. Robustness stability and local uniform stability of CNNs under perturbations were considered respectively [8,10]. The CNNs with time delays can be encountered in [11–15]. By using the nonsmooth analysis techniques and Lyapunov functional methods, the global exponential stability of delayed CNNs was guaranteed [11]. In [12], some conditions were obtained to ensure existence and global exponential stability of CNNs with multiple delays. The issue of global stability for CNNs with time-varying delay and discontinuous activation functions was discussed [14].

Although there are many studies dedicated to investigating the stability of CNNs, most of the investigations focused on the continuous-time systems or discrete-time systems. In reality, however, there are many real-world systems and neural processes that behave in piecewise continuous style interlaced. Consequently, it is meaningful to research continuous and discrete systems under the same framework. Correspondingly, the theory of time scales, which not only unifies the continuous-time and discrete-time domains but

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* Corresponding author. Tel.: +86 510 85707270; fax: +86 510 85910532.
E-mail addresses: czliuyanzi@yahoo.com (Y. Liu), yyq640613@gmail.com (Y. Yang).

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also "between" them, emerged as the times require. It was initiated by Hilger in 1988 [36], and has recently received a lot of attention. The book on time scales, by Bohner and Peterson [37], summarized and organized the time scale calculous theory in detail. Therefore, we recommend the interested readers to crack Reference [37].

As is well-known, time delays in stabilizing the negative feedback term have a tendency to destabilize a system. Therefore, the effect of typical time delays called leakage (or "forgetting") delays which is the first term in each of the right side of system (1) cannot be ignored. Recently, the impact of leakage delay in various neural networks has been studied by many researchers in the literature [27–34]. By using the methods of degenerate Lyapunov–Krasovskii functionals and inequalities together with some properties of M-matrices, sufficient conditions were derived to guarantee the stability of BAM with leakage delays in [27]. Based on the approach of the Lyapunov–Krasovskii functional with free-weighting matrices using linear matrix inequality (LMI), the stability of BAM FCNNs with leakage delays was discussed [28]. Besides, via using the Lyapunov functional method and differential inequality techniques, the stability of anti-periodic solutions for shunting inhibitory cellular neural networks [32], anti-periodic solutions for HCNNs [33] and almost periodic solutions for cellular neural networks [34] was investigated under the model with time varying delays in leakage terms. However, to the best of the authors′ knowledge, there are no results on the problem of stability for competitive neural networks with such delays so far, therefore, the authors have initiated investigations of the behavior of solution for these typological systems.

Recently, anti-periodic problem of nonlinear differential equations had arisen considerable attention [16–24]. Among them, existence and stability of anti-periodic solutions were concerned for BAM neural networks [16], Cohen–Grossberg neural networks [17], shunting inhibitory cellular neural networks [18] and high-order Hopfield neural networks [19] on time scales. However, all of the above-mentioned works on the anti-periodic solutions have not been considered for the CNNs on time scales. This motivates our present research. In this paper, we will discuss the existence and the global exponential stability of anti-periodic solution for CNNs with delays in the leakage terms on time scales. Such a model is described by the following form:

$$\begin{cases} STM: x_i^{\nabla}(t) = -\alpha_i(t)x_i(t-\delta_i) \\ \qquad + \sum_{j=1}^{N} D_{ij}(t)f_j(x_j(t)) + \sum_{j=1}^{N} D_{ij}^{\tau}(t)f_j(x_j(t-\tau_{ij})) \\ \qquad + B_i(t)S_i(t) + I_i(t) \\ LTM: S_i^{\nabla}(t) = -c_i(t)S_i(t-\varrho_i) + E_i(t)f_i(x_i(t)) + J_i(t) \end{cases} \tag{1}$$

with the initial

$$x_i(s) = \varphi_i(s), \quad s \in (-\tau_i, 0]_T, \tag{2}$$

$$S_i(s) = \phi_i(s), \quad s \in \left(-\varrho_i, 0\right]_T, \tag{3}$$

where $i,j = 1,\dots,N$; $x_i(t)$ is the neuron current activity level, $\alpha_i(t), c_i(t)$ is the time variable of the neuron and there exist positive numbers $\underline{\alpha}_i$, $\overline{\alpha}_i$ and $\underline{c}_i$, $\overline{c}_i$ such that $0 < \underline{\alpha}_i \le \alpha_i(\cdot) \le \overline{\alpha}_i$ and $0 < \underline{c}_i \le c_i(\cdot) \le \overline{c}_i$. $f_j(x_j(t))$ is the output of neurons, $D_{ij}(t)$ and $D_{ij}^{\tau}(t)$ represent the connection weight and the synaptic weight of delayed feedback between the ith neuron and the jth neuron respectively, $B_i(t)$ is the strength of the external stimulus, $E_i(t)$ denotes disposable scale, transmission delays τ_{ij} and leakage delays δ_i, ϱ_i are nonnegative constants and satisfy $0 < \delta_i, \tau_{ij} \le \tau_i$ (τ_i is a positive constant).

T is an ω-periodic time scale, and $\varphi_i(\cdot)$, $\phi_i(\cdot)$ are rd-continuous. For the sake of simplicity, set $[a,b]_T = \{t \in T, a \le t \le b\}$, and assume that $0 \in T$. T is unbounded above. We denote

$$\overline{x}_i = \max_{t \in [0,\omega]_T} |x_i(t)|, \quad \overline{S}_i = \max_{t \in [0,\omega]_T} |S_i(t)|, \quad \overline{\mu} = \max_{t \in [0,\omega]_T} |\mu(t)|,$$
$$\overline{D}_{ij} = \max_{t \in [0,\omega]_T} |D_{ij}(t)|, \quad \overline{D}_{ij}^{\tau} = \max_{t \in [0,\omega]_T} |D_{ij}^{\tau}(t)|, \quad \overline{B}_i = \max_{t \in [0,\omega]_T} |B_i(t)|,$$
$$\overline{E}_i = \max_{t \in [0,\omega]_T} |E_i(t)|, \quad \overline{I}_i = \max_{t \in [0,\omega]_T} |I_i(t)|, \quad \overline{J}_i = \max_{t \in [0,\omega]_T} |J_i(t)|.$$

Through this paper, we make the following assumptions:

(H_1): $\alpha_i(t)$, $c_i(t)$, $D_{ij}(t)$, $D_{ij}^{\tau}(t)$, $B_i(t)$, $E_i(t)$, $I_i(t)$, $J_i(t)$ are continuous functions. $D_{ij}(t)$, $D_{ij}^{\tau}(t)$, $E_i(t)$ are ω-periodic. They satisfy

$$\alpha_i\left(t+\frac{\omega}{2}\right) = \alpha_i(t), \quad c_i\left(t+\frac{\omega}{2}\right) = c_i(t), \quad B_i\left(t+\frac{\omega}{2}\right) = B_i(t),$$
$$I_i\left(t+\frac{\omega}{2}\right) = -I_i(t), \quad J_i\left(t+\frac{\omega}{2}\right) = -J_i(t),$$
$$D_{ij}\left(t+\frac{\omega}{2}\right)f_j(u) = -D_{ij}(t)f_j(-u), \quad D_{ij}^{\tau}\left(t+\frac{\omega}{2}\right)f_j(u) = -D_{ij}^{\tau}(t)f_j(-u),$$
$$E_i\left(t+\frac{\omega}{2}\right)f_j(u) = -E_i(t)f_j(-u),$$

for $i,j = 1,\dots,N$.

(H_2): The continuous function $f_i \in C(R,R)$ is a Lipschitz function, that is, there exists a positive constant $k_i > 0$, such that for all $x,y \in R$

$$|f_i(x) - f_i(y)| \le k_i|x-y|, \quad i = 1,\dots,N. \tag{4}$$

The organization of this paper is as follows. In Section 2, we introduce some necessary notations and lemmas. In Section 3, the existence of anti-periodic solution is proved by using the continuation theorem of coincidence degree theory. In Section 4, some sufficient conditions are obtained to guarantee the global exponential stability of the anti-periodic solution for CNNs. In Section 5, two examples are given to illustrate the effectiveness of the theoretical results. Finally a discussion follows in Section 6.

2. Preliminary

For the convenience, some definitions and lemmas are introduced.

Definition 1 (*Bohner and Peterson [37]*). A time scale T is an arbitrary nonempty closed subset of the real set R with the topology and ordering inherited from R.

The forward jump operator σ, the backward jump operator $\rho : T \to T$ and the graininess $\mu : T \to R^+$ are defined, respectively, by $\sigma(t) = \inf\{s \in T, s > t\}$, $\rho(t) = \sup\{s \in T, s < t\}$ and $\mu(t) = t - \rho(t)$. A point $t \in T$ is called right-scattered, right-dense, left-scattered, left-dense, if $\sigma(t) > t, \sigma(t) = t$, $\rho(t) < t$, $\rho(t) = t$ holds, respectively. A set T^k is introduced in addition to $T : T^k = T \backslash \{m\}$ if T has a left-scattered maximum m and $T^k = T$ otherwise.

In this paper, we only consider the unbounded time scales.

Definition 2 (*Bohner and Peterson [37]*). Let $\omega \in R$, $\omega > 0$, T is a periodic time scale if $t \in T$ then $t \pm \omega \in T$, for $T \ne R$, the smallest positive ω is called the period of the time scale T.

Definition 3 (*Bohner and Peterson [37]*). A function $f : T \to R$ is called right-dense continuous provided it is continuous at right-dense point of T and the left side limit exists (finite) at left-dense point of T. The set of all right-dense continuous functions on T is defined by $C_{rd} = C_{rd}(T) = C_{rd}(T,R)$.

Definition 4 (*Hong [35]*). For $f : T \to R$ and $t \in T^k$, we define the so-called ∇−derivative of f, $f^{\nabla}(t)$, to be the number (if it exists) with the property that, for any $\varepsilon > 0$ there is an N-neighborhood of t with

$$|f(s) - f(\rho(t)) - f^{\nabla}(t)(s - \rho(t))| \le \varepsilon|s - \rho(t)| \quad \text{for all } s \in N.$$

We call $f^{\nabla}(t)$ the nabla derivative of f at t.

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