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Exponential stability of Markovian jumping stochastic Cohen–Grossberg neural networks with mode-dependent probabilistic time-varying delays and impulses ☆

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ABSTRACT

This paper deals with robust exponential stability of Markovian jumping stochastic Cohen–Grossberg neural networks (MJSCGNNs) with mode-dependent probabilistic time-varying delays, continuously distributed delays and impulsive perturbations. By construction of novel Lyapunov–Krasovskii functional having the triple integral terms, the double integral terms having the positive definite matrices dependent on the system mode and MJSCGNNs system transformation variables, new delay-dependent exponential stability conditions are derived in terms of linear matrix inequalities (LMIs). By establishing a stochastic variable with Bernoulli distribution, the information of probabilistic time-varying delay is considered and transformed into one with deterministic time-varying delay and stochastic parameters. Furthermore, a mode-dependent mean square robust exponential stability criterion is derived by constriction of new Lyapunov–Krasovskii functional having modes in the integral terms, linear matrix inequalities and some stochastic analysis techniques. Finally, two numerical examples are provided to show the effectiveness of the proposed methods.

1. Introduction

The Cohen–Grossberg neural network (CGNN) model, proposed by Cohen and Grossberg [1] in 1983, has attracted considerable attentions due to their extensive applications in classification of patterns, associative memories, image processing, quadratic optimization and other areas. Over a decade, many scientific and technical workers have been joining the study fields with great interest, and various interesting results for CGNNs with/without delays have been reported [2–11]. Because time delays are often encountered in very large scale integration (VLSI) implementations of artificial neural networks due to delay transmission line and partial element equivalent circuit (PEEC), delayed neural networks (DNNs) have become a focus of research and a great number of results have been reported in the literature. As is well known, in real nervous systems, synaptic transmission is a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes [12]. On the other hand, it has been pointed out in [13] that a neural network could be stabilized or destabilized by certain stochastic inputs. Besides time delays, in the applications and designs of networks, some unavoidable uncertainties, which result from using an approximate system model for simplicity, parameter fluctuations, data errors, and so on, must be integrated into the system model. Such time delays, parametric uncertainties and stochastic disturbances may significantly influence the overall properties of a dynamic system. Therefore, it is of practical importance to study the stochastic effects on the stability property of delayed CGNNs [14–26].

Meanwhile, in a real system, time-delay often exists in a random form, i.e., some values of time-delay are very large but the probability taking such large values is very small, which will lead to some conservatism if only the information of variation range of time-delay is considered. Thus, recently, some researchers have considered the stability for various neural networks with probability-distribution delays [27–34]. Markovian jump systems, introduced by Krasovskii and Lidskii [35] in 1961, have received increasing attentions in the past years [36,37]. It should be noted that Markovian jump systems can be considered a special class of hybrid systems, which can be described by a set of linear systems with the transitions between models determined by a Markovian chain in a finite mode set. This kind of systems has applications in

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economic systems, modeling production systems and other practical systems. In this regard, a great number of results on stability analysis for neural networks with Markovian jumping parameters have been reported in the literature and various approaches have been proposed [38,39] and references therein.

On the other hand, impulsive effects exist widely in many evolution processes in which states are changed abruptly at certain moments of time, involving such fields as medicine and biology, economics, mechanics, electronics and telecommunications, see for example [40] and references therein. Thus, the study of impulsive neural networks with delays is a very good research topic in recent years and many researchers have investigated the problem of stability analysis of impulsive neural networks with delays [41,42]. Neural networks are often subject to impulsive perturbations that in turn affect dynamical behaviors of the systems [43]. Therefore, it is necessary to take impulsive effects into account on dynamical behaviors of neural networks [44–47]. To the best of the author's knowledge, very few results on the problem of exponential stability analysis for Markovian jumping stochastic Cohen–Grossberg neural networks with mode-dependent probabilistic time-varying delays and impulses have been studied in the literature. This motivates our present research.

Inspired by the above discussions, in this paper, the robust exponential stability results for Markovian jumping stochastic Cohen–Grossberg neural networks (MJSCGNNs) with mode-dependent probabilistic time-varying delays, continuously distributed delays and impulsive perturbations are considered. By constructing of novel Lyapunov–Krasovskii functional having the triple integral terms and introducing of free-weighting matrices, several new criteria for global exponential stability of MJSCGNNs are derived, which are expressed in terms of LMIs. Finally, the results are illustrated through some numerical simulation examples.

Notation: Let $\mathbb{R}^n$ denote the n-dimensional Euclidean space and the superscript "T" denote the transpose of a matrix or vector. I denotes the identity matrix with compatible dimensions. $diag(\cdots)$ denotes a block diagonal matrix. For square matrices, M_1 and M_2, the notation $M_1 > (\geq, <, \leq) M_2$ denotes $M_1 - M_2$ is a positive-definite (positive-semi-definite, negative, negative-semi-definite) matrix. $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ stand for the minimum and maximum eigenvalues of a given matrix. Let $(\Omega, \mathfrak{F}, \mathcal{P})$ be a complete probability space with a natural filtration $\{\mathfrak{F}_t\}_{t\geq 0}$ and $\mathbf{E}[\cdot]$ stand for the correspondent expectation operator with respect to the given probability measure $\mathcal{P}$. Also, let $\tau > 0$ and $\mathcal{C}([-\tau, 0]; \mathbb{R}^n)$ denote the family of continuously differentiable function ϕ from $[-d, 0]$ to $\mathbb{R}^n$ with the norm $\|\phi\| = \sup_{-d\leq\theta\leq 0}|\phi(\theta)|$, where $|\cdot|$ is the Euclidean norm in $\mathbb{R}^n$ and $d = \max\{d_2\}$. Denote by $\mathcal{C}^b_{\mathfrak{F}_0}([-d, 0]; \mathbb{R}^n)$ the family of bounded $\mathfrak{F}_0-$measurable, $\mathcal{C}([-d, 0]; \mathbb{R}^n)-$valued random variables $\xi = \{\xi(\theta) : -d \leq \theta \leq 0\}$ such that $\int_{-d}^{0} \mathbf{E}|\xi(\theta)|^2\, ds < \infty$.

2. Problem description and preliminaries

In this paper, the Markovian jump stochastic CGNNs with both impulsive perturbations and mixed time delays are described as follows:

$$\begin{cases} dx(t) = -\tilde{a}(x(t), r(t))[\tilde{b}(x(t), r(t)) - A(r(t))\tilde{f}(x(t)) - B(r(t))\tilde{g}(x(t-d(t))) \\ \qquad - C(r(t)) \int_{-\infty}^{t} K(t-s)\tilde{h}(x(s))\, ds]\, dt \\ \qquad + \sigma(x(t), x(t-d(t)), \int_{-\infty}^{t} K(t-s)\tilde{h}(x(s))\, ds, r(t))\, dw(t), \quad t \neq t_k, \\ x(t_k) = D_k(r(t))x(t_k^-), \quad t = t_k, \end{cases} \tag{1}$$

for $t \geq 0$ and $k = 1, 2, \ldots,$ where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T$ is the state vector associated with the n neurons, $\tilde{a}(x(t), r(t)) = diag(\tilde{a}_1(x_1(t), r(t)), \tilde{a}_2(x_2(t), r(t)), \ldots, \tilde{a}_n(x_n(t), r(t)))$ denotes the amplification function and assumed to be positive, bounded and locally Lipschitz continuous, $\tilde{b}(x(t), r(t)) = [\tilde{b}_1(x_1(t), r(t)), \tilde{b}_2(x_2(t), r(t)), \ldots, \tilde{b}_n(x_n(t), r(t))]$ denotes the appropriately behaved function such that the solution of the system given in (1) remains bounded. The matrices $A(r(t)) = (a_{ij}(r(t)))_{n\times n}, B(r(t)) = (b_{ij}(r(t)))_{n\times n}$ and $C(r(t)) = (c_{ij}(r(t)))_{n\times n}$ are the connection weight matrix, the time varying delay connection weight matrix, and the distributed delay connection weight matrix, respectively; $\tilde{f}(x(t)) = [\tilde{f}_1(x_1(t)), \tilde{f}_2(x_2(t)), \ldots, \tilde{f}_n(x_n(t))]^T$, $\tilde{g}(x(t)) = [\tilde{g}_1(x_1(t)), \tilde{g}_2(x_2(t)), \ldots, \tilde{g}_n(x_n(t))]^T$ and $\tilde{h}(x(t)) = [\tilde{h}_1(x_1(t)), \tilde{h}_2(x_2(t)), \ldots, \tilde{h}_n(x_n(t))]^T$ are the neuron activation functions. $w(t) = (w_1(t), w_2(t), \ldots, w_n(t))$ is an n-dimensional standard Brownian motion defined on a complete probability space $(\Omega, \mathfrak{F}, \mathcal{P})$. Moreover, we assume that the Brownian motion $\{w(t), t \geq 0\}$ is independent from the Markov chain $\{r(t), t \geq 0\}$. $K(t-s) = (K_{ij}(t-s))_{n\times n}$ and the delay kernel $K_{ij}(\cdot)$ is a real valued non-negative continuous function defined on $[0, \infty)$ and such that $\int_0^\infty K_{ij}(\theta)\, d\theta = 1$ for $i, j = 1, 2, \ldots, n$.

Let $\{r(t), t \geq 0\}$ is a right-continuous Markov chain on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$ taking values in a finite state space $\mathcal{S} = \{1, 2, \ldots, N\}$ with generator $\Gamma = (\delta_{ij})_{N\times N}$ given by

$$P\{r(t+\Delta t) = j | r(t) = i\} = \begin{cases} \delta_{ij}\Delta t + o(\Delta t), & i \neq j, \\ 1 + \delta_{ii}\Delta t + o(\Delta t), & i = j, \end{cases}$$

where $\Delta t > 0$ and $\lim_{\Delta t \to 0} o(\Delta t)/\Delta t = 0$, $\delta_{ij} \geq 0$ is the transition rate from i to j, if $i \neq j$ while $\delta_{ii} = -\sum_{j=1, j\neq i}^{N} \delta_{ij}$.

$D_k(r(t)) \in \mathbb{R}^{n\times n}, k \in \mathbb{Z}_+$ is the impulse gain matrix at the moment of time t_k. The discrete set $\{t_k\}$ satisfies $0 = t_0 < t_1 < \cdots < t_k < \cdots$, $\lim_{k\to\infty} t_k = \infty$. $x(t_k^-)$ and $x(t_k^+)$ denote the left-hand and right hand limits at t_k, respectively. Assume that $x(t)$ is right-continuous, i.e., $x(t_k^+) = x(t_k)$. $d(t) \geq 0$ denotes the time-varying delay and is assumed to satisfy $0 \leq d(t) \leq d_2$. To ensure the existence of a solution to (1), it is assumed that the time-varying delay $d(t)$ has a bounded derivative. In practice, there exists a constant d_1, where $0 \leq d_1 \leq d_2$, such that $d(t)$ takes values in $[0, d_1]$ and $(d_1, d_2]$ with certain probability. Therefore, $d(t)$ is a random variable which takes values in the intervals $[0, d_1]$ and $(d_1, d_2]$.

In this paper, the following assumptions on the neuron activation functions, amplification function, the behaved function, and probability distribution are made.

Assumption 2.1. The neuron activation functions $\tilde{f}_j(\cdot), \tilde{g}_j(\cdot)$ and $\tilde{h}_j(\cdot)$ satisfy $\tilde{f}_j(0) = \tilde{g}_j(0) = \tilde{h}_j(0) = 0$ and

$$\hat{u}_{1j}^- \leq \frac{\tilde{f}_j(x) - \tilde{f}_j(y)}{x - y} \leq \hat{u}_{1j}^+, \tag{2}$$

$$\hat{u}_{2j}^- \leq \frac{\tilde{g}_j(x) - \tilde{g}_j(y)}{x - y} \leq \hat{u}_{2j}^+, \tag{3}$$

$$\hat{u}_{3j}^- \leq \frac{\tilde{h}_j(x) - \tilde{h}_j(y)}{x - y} \leq \hat{u}_{3j}^+, \tag{4}$$

for all $x, y \in \mathbb{R}$, $x \neq y$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$. The constants $\hat{u}_{1j}^-$, $\hat{u}_{1j}^+$, $\hat{u}_{2j}^-$, $\hat{u}_{2j}^+$, $\hat{u}_{3j}^-$, $\hat{u}_{3j}^+$ in Assumption 2.1 are allowed to be positive, negative or zero.

Assumption 2.2. There exist positive constants a_{ij}^0, a_{ij}^1 $(i = 1, 2, \ldots, N, j = 1, 2, \ldots, n)$ such that

$$0 < a_{ij}^0 \leq \tilde{a}_j(x_j(t), i) \leq a_{ij}^1$$

for all $x_j(t) \in \mathbb{R}, r(t) = i, i \in S$ and $j = 1, 2, \ldots, n$.

Assumption 2.3. There exist positive constants $\overline{\zeta}_{ij}$ $(i = 1, 2, \ldots, N, j = 1, 2, \ldots, n)$ such that

$$x_j(t)\tilde{b}_j(x_j(t), i) \geq \overline{\zeta}_{ij} x_j^2(t)$$

for all $x_j(t) \in \mathbb{R}, r(t) = i, i \in S$ and $j = 1, 2, \ldots, n$.