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Guaranteed SLAM—An interval approach

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HIGHLIGHTS

- Present a new guaranteed SLAM methods using interval methods (i-SLAM).
- Prove the convergence of i-SLAM in the presence of nonlinear and non-Gaussian models.
- Present a comparison between i-SLAM and probabilistic SLAM.

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ABSTRACT

This paper proposes a new approach, *interval Simultaneous Localization and Mapping* (i-SLAM), which addresses the robotic mapping problem in the context of interval methods, where the robot sensor noise is assumed bounded. With no prior knowledge about the noise distribution or its probability density function, we derive and present necessary conditions to guarantee the map convergence even in the presence of nonlinear observation and motion models. These conditions may require the presence of some anchoring landmarks with known locations. The performance of i-SLAM is compared with the probabilistic counterparts in terms of accuracy and efficiency.

1. Introduction

Mobile robotic mapping has been an essential challenge in robotics where a robot needs an accurate map to navigate the environment. Simultaneous Localization and Mapping (SLAM) has been an active topic in robotics literature over the past three decades that explicitly addresses the mapping problem, and there has been a plethora of publications that proposed very efficient solutions [1–7]. These solutions are generally based on different assumptions about the SLAM problem, and they handle measurement uncertainties from noisy sensors using either (i) probabilistic approaches, or (ii) set-membership approaches and related interval methods.

The probabilistic methods for robotic mapping rely on studying the propagation of probabilistic distributions of the sensor noise and the unknown parameters, e.g., robot pose and landmark locations. These methods include: Extended Kalman Filter (EKF) SLAM [4], Factored Solution to the SLAM (FastSLAM) [6], and Graph SLAM [8]. In EKF SLAM, the *state* of the system corresponds to the joint distribution of robot pose and all landmarks in the environment at any timestep, and it is represented by a Gaussian distribution. The advantages of this approach lie in the fact that Kalman filter is quite simple to implement, and, like any Bayes filter, it is a recursive approach which is useful for online mapping applications. However, optimality and convergence of this approach are not guaranteed for nonlinear or non-Gaussian models [9] despite its good performance in practice.

SLAM problem has an interesting property where knowing the exact robot path makes landmark estimation statistically independent. Such property is exploited in FastSLAM [6] where Rao-Blackwellised particle filtering [10] is used to factor the SLAM problem into (i) localization problem that uses a Monte Carlo method or particle filter [11], and (ii) mapping problem that uses EKF. This approach is superior to EKF SLAM in terms of complexity and robustness to data association errors. But since EKF is used, convergence is not guaranteed for the nonlinear or non-Gaussian models [12,2].

Graph-based SLAM are several approaches that attempt to solve SLAM using *maximum likelihood estimation* (MLE) of the map given all the measurements in the context of graphical network. Lu-and-Milios algorithm [13] and GraphSLAM [8] are examples of such approaches to handle the data association problem with measurements from laser scanners.

Probabilistic SLAM methods have been used successfully in many practical applications and several hybrid approaches have

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been proposed to get the best qualities of each single method. However, they all depend on the fact that the noise can be described by some distribution with known probability density function. Moreover, convergence of maps using these techniques is only guaranteed in very special cases with strong assumptions such as Gaussian linear models. Most of real robotic systems are nonlinear, and many sensors do not possess Gaussian noise model or their distributions are simply not available. Hence, using probabilistic methods can lead to an approximate solution, i.e., not guaranteed.

Interval methods [14–16], on the other hand, do not require any prior knowledge about the probability distribution of uncertainties, except that they are bounded by some known real intervals. These methods have shown promising results in the field of parameter estimation [17,18], and they found their way to the mobile robotic applications, either in low-level control or high-level control. In low-level control, such as in robot locomotion, interval methods were used to study the stability of complex nonlinear robotic systems [19], to rigorously compute *capture domains* as defined in [20,21], or to synthesize nonlinear controllers as introduced in [22]. In terms of high-level control, interval methods were used for robot localization as presented in [23,14,24] where nonlinear robot models were used, such as motion model and observation model. In these applications, laser scanners were used with the assumption that the noise is bounded.

These methods use the set of measurements and construct a *constraint satisfaction problem* (CSP) [25,14] based on the robot motion and observation models. Then, a guided search is performed to find the smallest possible domain of the robot pose. Gning et al. developed *box particle filter* [26,27] that combines the advantages of interval methods and probabilistic particle filters as an attempt to reduce the number of required particles representing the posterior distribution of Bayes filter. This box particle filter approach is applied for tracking purposes and mobile robot localization. Another work which combines both probabilistic and interval methods was conducted in [28] to solve mobile robot localization problems.

In terms of robotic mapping, there have been several publications [3,29–31] that study mapping using interval methods, most of them use an approach similar to graph-based SLAM, but in the context of a constraint satisfaction problem with interval domains. For example, Jaulin in [3] used an underwater robot with sonar, and Vincke et al. in [29] used a ground robot with depth sensors to solve the SLAM problem with nonlinear models. By keeping track of all observations and control inputs, these approaches generate a set of constraints to be satisfied with respect to the robot pose and the landmark positions using constraints propagation. Moreover, interval range-only SLAM is introduced in [30] where a range sensor, such as sonar or lidar, is employed without the bearing information. This approach is applied to the SLAM problem in a manner similar to occupancy grid, however, the map resolution varies from one region to another depending on the size of obstacles and their geometric shapes. Bethencourt and Jaulin used a Kinect sensor and interval methods for the purpose of 3-D reconstruction using point features in [31], and they employed an IMU to estimate the sensor motion and assumed bounded noise. In the presence of faulty sensors or measurement outliers, applying interval methods directly will result in empty sets. In probabilistic methods, RANSAC [32] is an efficient approach that removes such outliers. The interval methods counterpart of RANSAC is *q-relaxed intersection* which is proven in to be evaluated in polynomial time as shown in [33]. Sliwka et al. [34] employed *q*-relaxed intersection to robustly localize an underwater robot using interval methods. Moreover, *q*-relaxed intersection was also used by De Freitas et al. [35] along with box particle filter [26] for robust tracking of a large number of objects simultaneously and in the presence of clutter measurements and possible outliers. This approach is beneficial for crowd tracking and monitoring applications.

The major disadvantage of interval methods is scalability with respect to the number of unknown variables. Jaulin et al. showed in [14] that using *contractors* [36] for solving CSPs can lead to polynomial time complexity in terms of evaluation. Such processing time, however, may not be sufficient for real time applications such as SLAM where the number of variables increases when new landmarks are observed. Thus, many interval methods are evaluated offline to obtain optimal solutions, though online applications are possible where suboptimal solutions are obtained [14]. Nonetheless, the results evaluated using interval methods guarantee that all possible solutions are found in terms of closed sets, unlike probabilistic Monte Carlo methods that return a finite subset of the solution [14].

Our contributions in this paper focus on the convergence of robotic mapping. More specifically, we aim to: (i) develop a new theory about compact sets in the context of real analysis, and use that to specify soft conditions that guarantee the convergence of robotic mapping for the case of nonlinear non-Gaussian models. These conditions may require the presence of some anchoring landmarks with known locations; (ii) introduce i-SLAM and employ interval methods to demonstrate the map convergence using the observation model only, and assuming that the data association problem is solved; and (iii) provide a comparison between probabilistic methods and interval methods with regard to robotic mapping, and demonstrate the performance of each using simulated data. Moreover, the proposed approach does not consider the robot motion model, though it can be incorporated in the problem formulation.

The paper is organized as follows: Section 2 introduces interval analysis and interval computations, followed by an illustration of efficient interval methods for solving generic constraint satisfaction problems. After that, i-SLAM is introduced in Section 3 where the robotic mapping problem is defined in the framework of constraint satisfaction problems, and interval methods algorithms are used to obtain the map estimate. Section 4 presents theoretical background that highlights convergence conditions for the mapping problem in the context of real analysis. Then, simulation and results are presented in Section 5 with a mobile robot moving in 2-D environment and using only measurement model. Also, this section presents a comparison between probabilistic mapping and interval mapping, and it highlights the advantages of each method in terms of performance and output quality. Finally, Section 6 concludes the paper with a discussion about the results, and Section 7 presents several areas of improvements that can be addressed in the future.

2. Interval analysis and methods

2.1. Real interval arithmetic and functions

A real interval is a connected, closed set in $\mathbb{R}$, and it is denoted by $[x] = [\underline{x}, \overline{x}]$. The two numbers $\underline{x}, \overline{x} \in \mathbb{R}$ represent the lower bound and upper bound of $[x]$, respectively. Basic arithmetic operations can be applied to intervals. Let $\diamond$ be a binary arithmetic operation, i.e., $\diamond \in \{+, -, *, /\}$, and $[x]$, $[y]$ are real intervals, then, interval operation is defined as follows:

$$[x] \diamond [y] = [\{x \diamond y \in \mathbb{R} \mid x \in [x], y \in [y]\}], \tag{1}$$

where $[\ \cdot\ ]$ is the *hull* operator [14]. Similarly, elementary functions can be extended to intervals. Consider a function $f : \mathbb{R} \to \mathbb{R}$, its interval counterpart is defined as:

$$[f]([x]) = [\mathbb{R} \cap \{f(x) \mid x \in [x]\}]. \tag{2}$$

Intervals can be extended to higher dimensions by introducing the *vector of intervals*, also called a *box*, and it is denoted by $[\mathbf{x}]$. A box is a compact subset of $\mathbb{R}^n$ that is defined as the Cartesian product of n closed intervals such that $[\mathbf{x}] = [x_1] \times [x_2] \times \cdots \times [x_n]$. The set of all n-dimensional boxes is denoted by $\mathbb{IR}^n$ [14].