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Real-time contact force distribution using a polytope hierarchy in the grasp wrench set

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HIGHLIGHTS

- An algorithm is proposed to compute a hierarchy of polytopes in the grasp wrench set.
- The polytope hierarchy provides a list of facets from the interior of the grasp wrench set to its boundary.
- The minimum contact force for grasping can be quickly computed by searching the facet list.
- The online computation of contact force distribution needs less than one millisecond on a normal PC.

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ABSTRACT

This paper presents an efficient approach to contact force distribution, which is aimed at computing optimal contact forces to generate the required wrench for grasping an object. It has been derived in the previous work that this problem can be reduced to computing the intersection of the ray originating from the origin along the required wrench with the boundary of the grasp wrench set. Noticing that the grasp wrench set is fixed once contact positions are determined, we propose an algorithm to pre-compute a hierarchy of polytopes in the grasp wrench set and a list of facets from the interior of the grasp wrench set to its boundary. Then, the ray's intersection can be quickly found by searching the list of facets and optimal contact forces can be computed in real time. Numerical examples show that the online computation of the proposed approach is one order of magnitude faster than the latest algorithm to compute the ray's intersection and two orders of magnitude faster than general-purpose optimization algorithms. This approach to contact force distribution is an iterative solution that can run until reaching a desired accuracy.

1. Introduction

While a multifingered robot hand is used to grasp an object, it is required to determine the force that each contact applies to the object such that the resultant wrench from all the contact forces resists the other wrenches applied to the object. This problem is known as contact force distribution (CFD). Due to the existence of multiple contacts and the underdetermined nature of the problem, there is an infinite number of distributions of forces among contacts that produce the same resultant wrench. This provides the freedom to optimize the contact forces based on certain criteria, such as minimizing their magnitudes or inclination angles. Moreover, each contact force must satisfy the friction constraint to prevent undesired motions at contact. Hence, the CFD problem is essentially a constrained optimization problem.

In the previous work, a majority of efforts have been spent on formulating the CFD problem as optimization problems in special forms that facilitate the use of the existing optimization algorithms. By replacing the quadratic friction cone with a polyhedral cone, the friction constraint can be written as a set of linear inequalities so that the resulting optimization problem becomes a linear program [1,2] or a quadric program with linear inequality constraints [3–5]. Also, it was observed that the friction constraint is equivalent to the positive definiteness of a matrix [6]. Based on this observation, the CFD problem was formulated as a semidefinite programming problem with linear matrix inequalities [7–9], which can be more efficiently solved by the existing convex programming algorithms. In order to further expedite a general-purpose optimization algorithm applied to this problem, the structure of constraints and the selection of initial conditions, step size, and stopping criteria were also explored in the above works. Comparative tests of these algorithms were reported in the work [10]. More recently, Gazeau et al. [4] reduced the CFD problem into a minimal distance problem and solved it with a gradient projection method. Cornellà et al. [5] proposed a similar formulation but solved its dual form instead of the primal form.

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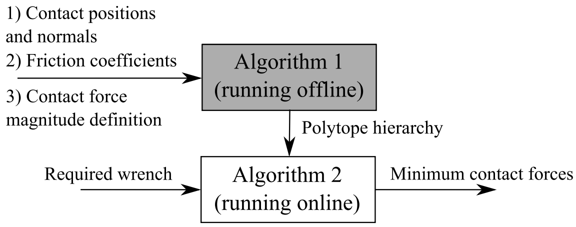

Fig. 1. Diagram of the proposed approach to contact force distribution (CFD). The three inputs to Algorithm 1 are assumed to be already known and Algorithm 1 pre-computes a hierarchy of polytopes in the grasp wrench set. Using the polytope hierarchy, Algorithm 2 computes the minimum contact forces with respect to the required wrench, which is dynamically changing, for manipulating the grasped object.

Boyd and Wegbreit [11] proposed an interior-point algorithm to compute CFD as a second-order cone problem.

Another category of approaches to CFD is based on the discovery that computing the minimum contact forces can be reduced to a geometric problem called ray-shooting [2], which computes the intersection of a ray with the boundary of a convex set. Specific geometric algorithms were proposed for this problem and shown to possess higher computational efficiency than general-purpose optimization algorithms to compute CFD [12,13].

Aiming to the real-time applicability, some approaches divide the entire computation of contact forces into offline and online phases and try to pre-calculate certain quantities in the offline phase that can facilitate the online computation of final contact forces [14–17].

The approach proposed in this paper falls in the last category and is intended for the case where the contact positions are known offline and the contact forces need to be computed online with respect to the required wrench for manipulating the grasped object to change its position and orientation without changing the contact positions. It follows from the discovery that the CFD problem can be reduced to computing the intersection of a ray with the boundary of the grasp wrench set [2]. Since the contact positions are known and do not change later, the grasp wrench set is fixed. Then, this approach first offline calculates a hierarchy of polytopes with a list of facets from the interior of the grasp wrench set to its boundary. By doing this, the ray's intersection can be quickly found online by searching the facet list and the minimum contact forces can be computed in real time. A diagram of the proposed approach is shown in Fig. 1. By this approach, the online computation takes less than one millisecond on a normal PC, which is one to two orders of magnitude faster than the geometric algorithms [12,13] or general-purpose optimization algorithms that need perform online iteration to compute the ray's intersection or minimum contact forces. The accuracy of the resulting minimum contact forces is determined by the established polytope hierarchy, which is computed by an iterative procedure to gradually expand a polytope inside the grasp wrench set. The procedure can iterate until the final expanded polytope is sufficiently close to the boundary of the grasp wrench set such that the final result can reach a desired accuracy.

The rest of this paper is organized as follows. Section 2 introduces the CFD problem and related results from the previous work. Section 3 presents our approach followed by numerical examples in Section 4. Section 5 concludes this paper.

2. Preliminaries

2.1. Basic knowledge and problem statement

Assume that a multifingered robot hand grasps an object by m contacts. In order to resist the other wrenches (i.e., forces and moments) applied to the object and maintain it in the mechanical equilibrium, the resultant wrench $\boldsymbol{w}_{\text{res}}$ from all the contact forces must be the negative of the external wrench $\boldsymbol{w}_{\text{ext}}$ (sum of the other wrenches) [18], which can be written in the object coordinate frame as

$$\boldsymbol{w}_{\text{res}} = \sum_{i=1}^{m} \boldsymbol{G}_i \boldsymbol{f}_i = -\boldsymbol{w}_{\text{ext}}, \tag{1}$$

where $\boldsymbol{G}_i = \begin{bmatrix} \boldsymbol{n}_i & \boldsymbol{o}_i & \boldsymbol{t}_i & \boldsymbol{0} \\ \boldsymbol{p}_i \times \boldsymbol{n}_i & \boldsymbol{p}_i \times \boldsymbol{o}_i & \boldsymbol{p}_i \times \boldsymbol{t}_i & \boldsymbol{n}_i \end{bmatrix}$ is the matrix converting the contact force $\boldsymbol{f}_i$ at contact i ($i = 1, 2, \dots, m$) into the wrench with respect to the object coordinate frame, $\boldsymbol{p}_i$ is the contact position vector, and $\boldsymbol{n}_i, \boldsymbol{o}_i, \boldsymbol{t}_i$ are the unit normal and two orthogonal unit tangent vectors at contact i with respect to the object coordinate frame. We consider the soft-finger contact model [18,19], for which the contact force $\boldsymbol{f}_i = [f_{i1}\ f_{i2}\ f_{i3}\ f_{i4}]^T$ comprises four components, namely three pure force components f_{i1}, f_{i2}, f_{i3} along $\boldsymbol{n}_i, \boldsymbol{o}_i, \boldsymbol{t}_i$, respectively, and a spin moment f_{i4} about $\boldsymbol{n}_i$. To maintain a stable contact, $\boldsymbol{f}_i$ must stay within the friction cone [18]

$$F_i \triangleq \left\{ \boldsymbol{f}_i \in \mathbb{R}^4 \mid f_{i1} \geq 0,\ \sqrt{\frac{f_{i2}^2 + f_{i3}^2}{\mu_i^2} + \frac{f_{i4}^2}{\mu_{si}^2}} \leq f_{i1} \right\}, \tag{2}$$

where μ_i and μ_{si} are the tangential and torsional friction coefficients, respectively. The soft-finger contact model is more mathematically general such that the rigid contact model with or without friction can be treated as its special cases. Thus, the approach proposed in this paper can be naturally applied to grasps with rigid contacts.

The goal of contact force distribution is to compute contact forces $\boldsymbol{f}_i \in F_i, i = 1, 2, \dots, m$ to generate the required wrench $\boldsymbol{w}_{\text{res}}$ in (1). Since there are usually more contacts than necessary, there is an infinite number of feasible solutions for $\boldsymbol{f}_i$ and the contact forces can be distributed such that their magnitude is minimal. It should be noted that the last force component f_{i4} is a spin moment and has a different unit from the other force components f_{i1}, f_{i2}, f_{i3}, which are pure forces. Then, the Euclidean norm of $\boldsymbol{f}_i$ would mix quantities with different units and does not make physical sense. Therefore, the magnitude of a contact force $\boldsymbol{f}_i$, denoted by $\|\boldsymbol{f}_i\|$, is defined as one of the following two quantities, which essentially are the infinity norm and a weighted norm of $\boldsymbol{f}_i$, respectively:

$$\|\boldsymbol{f}_i\| \triangleq f_{i1}, \tag{3a}$$

$$\|\boldsymbol{f}_i\| \triangleq \sqrt{f_{i1}^2 + f_{i2}^2 + f_{i3}^2 + \mu_i^2 f_{i4}^2 / \mu_{si}^2}. \tag{3b}$$

Since the other force components have been constrained by f_{i1} in the friction constraint (2), most of the previous work chose the definition (3a), which is also easier to calculate than the nonlinear formulation (3b). The overall magnitude of contact forces $\boldsymbol{f}_i$, $i = 1, 2, \dots, m$ is often defined in one of the following two forms, i.e., the sum or the maximum of normal force components [6,8,11]

$$\sigma \triangleq \sum_{i=1}^{m} \|\boldsymbol{f}_i\|, \tag{4a}$$

$$\sigma \triangleq \max_{i=1,2,\dots,m} \|\boldsymbol{f}_i\|. \tag{4b}$$

From (3) and (4) there are actually four different definitions of overall contact force magnitude. The approach proposed in this paper can work with any of those, as shown in Section 2.2.

2.2. Results from the previous work

It has been derived that the computing of $\boldsymbol{f}_i \in F_i$, $i = 1, 2, \dots, m$ with the minimum magnitude σ^* can be reduced to computing the intersection of the ray R originating from the origin