Robotics and Computer–Integrated Manufacturing 000 (2017) 1–10

Contents lists available at ScienceDirect

Robotics and Computer–Integrated Manufacturing

journal homepage: www.elsevier.com/locate/rcim

Robust point-to-point trajectory planning for nonlinear underactuated systems: Theory and experimental assessment

Paolo Boscariol*, Dario Richiedei

Dipartimento di Tecnica e Gestione dei sistemi Industriali - DTG Università degli Studi di Padova, Stradella S. Nicola 3, 36100 Vicenza, Italy

A R T I C L E I N F O

Keywords:
Model-based trajectory planning
Smooth trajectory
Trajectory planning
Underactuated system

A B S T R A C T

This paper proposes the theory and the experimental assessment of a robust model-based trajectory planning algorithm for underactuated nonlinear systems in point-to-point motion. The method has been developed to increase the insensitivity of the resulting trajectory to parametric uncertainties of the plant. The proposed method is based on an augmented model that considers an approximate dynamics of the servo-controlled axis driving the actuated degrees of freedom. Trajectory planning is accomplished by computing the motion reference for the actuated degrees of freedom to reduce the effects of the uncertainty on the dynamic model. By exploiting an indirect variational formulation method, the necessary optimality conditions deriving from the Pontryagin's minimum principle are imposed, thus leading to a differential Two-Point Boundary Value Problem (TPBVP). Numerical solution of the latter is accomplished by means of collocation techniques to handle model nonlinearities.

Robustness is achieved by including additional conditions on the sensitivity functions for the initial and final points of the trajectory. The experimental evaluation of the effectiveness of the proposed method is performed on a double-pendulum crane, by comparing the transient and residual vibration. A comparison is provided with three well-established input-shaping methods, and robustness against unmodeled parametric perturbations and tracking errors is evaluated. The experimental evidence indicates that the inclusion of the additional constrains results in an effective reduction of the residual vibration, and that the proposed method is well suited to perform high speed motion.

1. Introduction

High-speed operation of robotic manipulators and automatic machines requires the concurrent use of effective control systems and smooth trajectories to ensure accurate tracking of the desired reference with minimum vibration excitation. The problem of motion-induced vibration is exacerbated in underactuated systems, such as cranes or machines with a lightweight construction of either the links or the joints [1,2]. Such manipulators might incur in severe vibration both during the motion and after motion completion, thus limiting their operativeness and their precision [3].

The literature proposes several approaches to improve the dynamic behavior of underactuated systems. Focusing on trajectory planning can reduce the need for accurate, high-bandwidth sensors devoted to vibration control. This feature is useful in those industrial applications where the use of additional sensor can be impractical or non cost-effective, as well as those where the system is controlled by standard, closed and proprietary industrial controllers. This fact has been attracting a lot of attention in the scientific community and among industrial practitioners.

The literature on trajectory planning algorithms is therefore quite extensive, as shown by the review paper [4]. A first distinction can be made between model-free and model-based approaches. The first approach is a general strategy that has the advantage of allows the same technique to be applied to several different machines without any knowledge of their dynamic model.

Model-free trajectory planning algorithms are often based on geometric approaches, and therefore they focus on the definition of time laws defined either in the joint space or in the operational space [5] using interpolation techniques. Vibration reduction is achieved by reducing or eliminating jerk peaks, which are responsible for the excitation of the mechanical structure of the machine [6]. Acceleration continuity and jerk limitation is obtained by choosing suitable motion primitives, such as B-splines [7,8] or cubic splines [9]. Their effectiveness is however limited to the possibility of reducing vibration excitation, since in general they cannot guarantee zero residual vibration.

In contrast, model-based approaches requires an adequate knowledge of the dynamics of the model for which the trajectory is planned. Therefore, they can generally lead to more accurate results and zero

* Corresponding author.
E-mail address: paolo.boscariol@unipd.it (P. Boscariol).

https://doi.org/10.1016/j.rcim.2017.10.001
Received 6 June 2017; Received in revised form 9 October 2017; Accepted 9 October 2017
Available online xxx
0736-5845/

residual vibrations, at the cost of a lesser robustness to model-plant mismatches, unless uncertainty is properly tackled in the design. A model can be used in the design by following several approaches. For example, zero residual vibration can be achieved by precise timing of motion laws such as s-curve speed profiles [10] or smoothed jerk profiles [11], by taking advantage of the knowledge of frequencies and damping factors of the main vibrational modes.

The same concept is exploited also for the class of methods referred to as input shaping, which have gained a wide diffusion [12,13] due to their effectiveness and simple implementation and are often used as benchmarks in the literature. Input shaping filters can be used to perform rest-to-rest motion with zero residual vibration for single mode [14] and multi-mode systems [15].

Input shaping techniques have also been extended to react to uncertainty or changes of the oscillation frequencies, leading to the definition of robust shapers and to the extra-insensitive robust shapers [16].

Similar performances levels can be achieved by translating the motion profile design into a filter design problem, that are used to produce smooth motion profiles when convolved with rough reference signals (see e.g. [17,18]).

An alternative approach to model-based trajectory planning is based on translating it as the solution of an optimal control problem. Among the extensive literature on such a topic, a main distinction can be made into direct and indirect optimization methods.

In the case of direct methods, the original optimal control problem is converted into a parameter optimization problem [19], by a proper discretization of robot kinematic variables. Then this new finite-dimensional problem can be solved through a wide number of efficient optimization algorithm, either deterministic or stochastic ones. For example, an optimization problem is solved in [20] to compute the coefficients which define the correct zero residual vibration as a combination of polynomial and cycloidal functions. In such a work, as wells as in others using direct methods such as [21], the solution of the optimization problem imposes extensive dynamic simulations and therefore they require a non-negligible computational effort. Hence, their effectiveness is reduced in the presence of systems with large number of degrees of freedom [22].

Indirect methods make use of calculus of variations: the necessary conditions of the Pontryagin's Minimum Principle (PMP) are imposed and the resulting Two-Point Boundary Value Problem (TP-BVP) is solved. Indirect methods are widely reckoned to be very accurate, particularly in the case of high degree of underactuation or multi-objective optimization [23]. Their application is very widespread in literature, and they have used for mobile robot application [24], flexible-joints robots [25], flexible-link robots [26] and cable-based robots [27], just to cite a few examples.

The general framework of calculus of variations can be adapted to include countless options in the optimization problem, and to account for constraints as well, which are always useful when addressing practical implementations.

One of the main drawbacks of this approach, that is inherited from its roots in optimal control, is the limited robustness to parametric mismatches between the plant used for the planning and the actual plant. Robustness to such changes is, instead, a highly useful characteristic that should be achieved by any control scheme or trajectory planning algorithm [28]. Therefore, the robustness issued has been tackled extensively in the field of closed-loop control, as testified by a literature too vast to be referenced in this work, but to the best of authors knowledge, there are very few works that specifically focus on robust trajectory planning algorithms.

One example is [29], in which robustness is achieved by introducing in the cost function a term of Gaussian cumulative noise. The work by Shin [30] focuses on the definition of robot trajectories by taking into account the uncertainties brought by payload variations trough the change of bounds on joint torques. Other interesting approaches to robust trajectory planning are currently available as solutions to the problem of robust optimization for dynamic systems: an extensive overview of this problem is available in [31]. One of the Authors of the present paper has recently proposed an extension of the variational approach to trajectory planning problems that can cope with parametric uncertainties [32], that unlike other methods in literature (such as those in [33,34]), can cope with plants described by nonlinear dynamics.

Model-based techniques casting optimal motion planning as an optimal control problem can be also grouped in accordance with the variables obtained as the output. For example, papers [35–37] compute the optimal profile of the command force (or torque) driving the actuated degrees of freedom, by casting, in practice, optimal motion planning as an inverse dynamics problem. A less common approach is, instead, performing optimal trajectory planning by synthesizing the optimal position (or speed) reference of the actuated degrees of freedom, such as the one in [38–40]. Although such an approach has attracted less attention, it has some practical advantages that make it suitable for the implementation in industrial robots or manipulators, as well as in complicate multibody systems. Indeed, it does not require estimating the control force needed to perform the motion, which are instead computed by the real-time feedback and feedforward control of the axis driving the underactuated system.

Experimental validation of the method is also provided through a double pendulum crane system, which is a three degree of freedom system described by a set of nonlinear differential equations. The experimental testbed is developed through an industrial robot with proprietary and closed controller, thus corroborating the ease of implementation in real systems. A comparison with three widely used input shaping techniques is provided as well.

2. System model formulation

The equations of motion of a multibody system with n degrees of freedom (dofs) can be written, given a proper choice of n independent coordinates $\mathbf{q}$, as the set of n nonlinear ordinary differential equations:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} = \mathbf{K}(\mathbf{q}) + \mathbf{G}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{B}(\mathbf{q})\mathbf{F} \tag{1}$$

$\mathbf{M} \in \Re^{m\times n}$ is the mass matrix, $\mathbf{K} \in \Re^{n}$ is the vector of position-dependent forces, i.e. elastic and gravity forces. Vector $\mathbf{G} \in \Re^{n}$ takes into account the gyroscopic and the centrifugal forces, as well as the damping forces. $\mathbf{B} \in \Re^{n\times m}$ is the force distribution vector, which weights the effect of the external control forces $\mathbf{F} \in \Re^{m}$. If $m < n$ the system is said to be underactuated, i.e. the number of the control forces is less than the size of the vector of the generalized coordinates. Hence $\mathbf{B}$ cannot be inverted. The dynamic model in Eq. (1) can be conveniently partitioned to highlight the contributions related to the m actuated coordinates $\mathbf{q}_a$ and the $n - m$ unactuated ones $\mathbf{q}_u$:

$$\begin{bmatrix} \mathbf{M}_{aa} & \mathbf{M}_{au} \\ \mathbf{M}_{au}^T & \mathbf{M}_{uu} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_a \\ \ddot{\mathbf{q}}_u \end{bmatrix} = \begin{bmatrix} \mathbf{K}_a(\mathbf{q}) \\ \mathbf{K}_u(\mathbf{q}) \end{bmatrix} + \begin{bmatrix} \mathbf{G}_a(\mathbf{q}, \dot{\mathbf{q}}) \\ \mathbf{G}_u(\mathbf{q}, \dot{\mathbf{q}}) \end{bmatrix} + \begin{bmatrix} \mathbf{B}_a \\ 0 \end{bmatrix} \mathbf{F} \tag{2}$$

This partition shows that the motion of the unactuated coordinates is determined by the motion of the actuated ones as:

$$\ddot{\mathbf{q}}_u = \mathbf{M}_{uu}^{-1}\left(\mathbf{K}_u + \mathbf{G}_u\right) - \mathbf{M}_{uu}^{-1}\mathbf{M}_{au}^T\ddot{\mathbf{q}}_a \tag{3}$$

Conversely, the actuated coordinates can be forced to follow a prescribed trajectory $\mathbf{q}_a^{ref}$ by choosing a proper control force profile, which is usually determined by the control scheme adopted. The availability of high-bandwidth closed loop control schemes, together with feedforward actions, can boost correct tracking of the desired trajectory for the actuated dofs. Under these circumstances, as often done in literature (see e.g. [2,41,42], just to mention a few notable examples), perfect tracking can be assumed, i.e. the actual trajectory is assumed to be equal to the planned one: $\mathbf{q}_a(t) = \mathbf{q}_a^{ref}(t)$.

An improved approach is instead proposed in this paper by assuming a dynamic relationship between the desired and the actual trajectories of the actuated dofs, that might be expressed through function $\mathbf{h}$:

$$\ddot{\mathbf{q}}_a(t) = \mathbf{h}(\ddot{\mathbf{q}}_a^{ref}(t)) \tag{4}$$