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Stiffness analysis of parallelogram-type parallel manipulators using a strain energy method

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ABSTRACT

Although stiffness analyses of specific parallelogram-type parallel manipulators (PTPMs) have been presented by several researchers, an algebraic expression is still needed to obtain the stiffness of a general PTPM. To address this issue, this paper uses a strain energy method considering the compliances of the mobile platform, the limb and the actuator of a PTPM. In this method, the deformation of the mobile platform, which has typically been ignored by many researchers, is integrated in the total deformation of the PTPM. After comparison with a FEA method, it is found that the proposed algebraic method is a comparable alternative to the FEA method to be used in the pre-design stage. Additionally, a new stiffness index is proposed to evaluate the stiffness property. Compared with other stiffness indices, the new index uses virtual work to unify the units of translation and orientation and relates the index value to the direction of the wrench experienced by a parallel manipulator in a task. With this index, the resistance of a PTPM to deformation under a given wrench can be measured easily.

1. Introduction

A parallelogram structure was first used [1] in 1980s to construct a purely translational parallel manipulator, named as Delta, which has been applied for pick-and-place operation in the industry. The orientation of the mobile platform of the Delta robot is well constrained by the parallelogram structure. After the Delta robot, the parallelogram structure was adopted by several researchers to construct the limbs of the triaglide [2] and the orthoglide [3], by replacing the rotary actuators of the Delta robot with translational actuators. In this paper, the Delta robot, the triaglide and the orthoglide are grouped as parallelogram-type parallel manipulators (PTPMs). A PTPM is defined to be a purely translational parallel kinematic manipulator which has three independent limbs and each of them is a parallelogram structure.

Stiffness is related to the accuracy of a manipulator since stiffness reflects the direct mapping between the externally applied wrench and the deformation of the manipulator. Although parallel manipulators present good performance in terms of accuracy and rigidity, it is still necessary to consider the stiffness in the pre-design stage, as the stiffness is dependent on the material property, the structural configuration and its dimension. Stiffness analysis of parallel manipulators attracts constant attention of researchers. Generally, the analysis methods fall into three categories, namely, experimental methods, finite element analysis (FEA) methods and algebraic methods.

Experimental methods are recommended to validate the mechanical design of a robotic system. Nevertheless, it is still challenging to set up a precise experimental configuration to investigate the stiffness of a multi-body robot, such as a parallel manipulator. The investigation of the stiffness is usually achieved by measuring the displacements of the manipulator under an external wrench. In practice, the displacements are attributed to the deformation of the manipulator, the clearance between connected components and the backlash of the actuators. The clearance and backlash cannot be avoided due to manufacturing and assembly errors. Therefore, the accuracy of experimental methods cannot be guaranteed. To decrease the effects of the clearance and backlash, Aginaga [4] applied an external force in the positive and negative directions consecutively to obtain an average result, although the author admitted that the error sources were not excluded successfully in the experiments. Applying a preload on a parallel manipulator is another method to reduce the clearance and backlash. This method was adopted by Huang [5] and Pinto [6]. Although the preload is able to decrease the clearance and backlash, it is difficult to determine an appropriate magnitude of the preload. Hence, the experimental methods are capable of obtaining the total displacement of a parallel manipulator under an external wrench, but it is difficult to isolate the deformation of a parallel manipulator from the source errors.

FEA methods are alternatives to the experimental methods.

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Nomenclature

PTPMs	parallelogram-type parallel manipulators
FEA	finite element analysis
$\{O\}$	global coordinate frame
$\{O'\}$	local coordinate frame
$R_{a,i}$	distance of an actuator to the origin of the global coordinate frame
$R_{b,i}$	distance of a limb to the origin of the local coordinate frame
l_i	length of a parallelogram limb
n_i	width of a parallelogram limb
d_i	distance of a limb to the motor driving the limb
α_i	angle between the guide trace of a translational actuator and the base plate
$\boldsymbol{A}$	inverse compliant Jacobian matrix
$\boldsymbol{C}$	overall compliance matrix
$\boldsymbol{K}$	overall stiffness matrix
$\boldsymbol{W}$	applied external wrench
$\delta\boldsymbol{\xi}$	infinitesimal twist
$\delta\boldsymbol{\chi}$	infinitesimal translation
$\delta\boldsymbol{\psi}$	infinitesimal rotation
U	strain energy
A	area of a cross section
E	elastic modulus
G	shear modulus
I	area moment of inertia
J	polar moment of inertia
k_{tor}	torsional stiffness of a motor
N	transmission ratio of a gear box
F_x	force in the x direction of the global coordinate frame
F_y	force in the y direction of the global coordinate frame
F_z	force in the z direction of the global coordinate frame
M_x	moment about the x axis of the global coordinate frame
M_y	moment about the y axis of the global coordinate frame
M_z	moment about the z axis of the global coordinate frame
δx	translation in the x direction of the global coordinate frame
δy	translation in the y direction of the global coordinate frame
δz	translation in the z direction of the global coordinate frame
δM_x	rotation about the x axis of the global coordinate frame
δM_y	rotation about the y axis of the global coordinate frame
δM_z	rotation about the z axis of the global coordinate frame
VW	virtual work stiffness index

With appropriate settings, modeling and meshing, FEA methods are able to obtain accurate results. Therefore, the FEA methods are adopted by many researchers to evaluate their analytical results [7–12]. However, the FEA methods are typically time-consuming [13]. Since the stiffness is dependent on the configuration and dimension of a parallel manipulator, the FEA methods always require a complete re-meshing and re-calculation if the configuration or the dimension is changed. This disadvantage can generate a huge computation load in the optimization design stage.

Compared with the FEA methods, algebraic methods deduce the stiffness of a parallel manipulator using algebraic expressions. With the algebraic expressions, it is easy to obtain the stiffness matrix even if the configuration or the dimension has been changed. However, algebraic methods always require several assumptions to be made. Gosselin [14] used a Jacobian matrix, which relates the velocity of the mobile platform of a parallel manipulator to the velocity of the actuator, to quantify the stiffness matrix. This quantification considered the compliance of the actuator, while the other components were assumed to be rigid. Several researchers [9,11] considered the compliance of the limb of a parallel manipulator with the other rigid components. El-Khasawneh [12] integrated the compliance of the limb and the compliance of the actuator into the stiffness analysis of a Stewart platform. The compliances of the limb and the actuator drew much research attention [4,5,7,15–18]. Cheng [7] found that the deformation of a parallel manipulator using a FEA method for stiffness analysis is larger than that obtained using an algebraic method, in which the actuator and the limb are assumed to be flexible. Based on this finding, Cheng mentioned that the difference might be caused by neglecting deformation of the mobile platform and passive joints. Rezaei [8] first considered the compliances of the mobile platform, the limb and the actuator to analyze the stiffness of a parallel manipulator. The manipulator uses three translational actuators. The compliance of the motor in the actuator was included in Rezaei's algebraic model. Nevertheless, the deformation of the lead screw in the actuator was neglected.

The stiffness analysis always derives a stiffness matrix. In the optimization stage, the stiffness matrix is required to be converted to a stiffness index to evaluate the stiffness quality of a parallel manipulator. Generally, the maximum or minimum eigenvalue of the stiffness matrix is used as the stiffness index [19–21], since the maximum and minimum eigenvalues present the most and least rigid values in the directions specified by the corresponding eigenvectors. It should be noted that the maximum and minimum eigenvalues are used to define upper and lower bounds of the stiffness. They should be evaluated together and cannot be combined to form a single index. Besides the eigenvalues, the determinant of the stiffness matrix [20], the condition number of the stiffness matrix [22], and the Euclidean norm of the diagonal elements of the stiffness matrix [23] can be accepted as stiffness indices. Since the translation and orientation of a parallel manipulator have different units, these indices cannot be interpreted easily. Additionally, these stiffness indices fail to relate the stiffness property of a parallel manipulator to the direction of a wrench applied on it. This relationship is important as a parallel manipulator presents different stiffness properties in different directions.

Although the stiffness analysis of the Delta robot, triaglide and orthoglide has been presented by several researchers, there is a lack of a general algebraic method for providing an algebraic expression for the PTPMs. More importantly, the reported algebraic methods generally ignore the deformation of the mobile platform. This paper uses the strain energy method to deduce a general algebraic stiffness matrix of the PTPMs. The deformation of the mobile platform, the limb, the motor and the lead screw (if used) is considered in the stiffness analysis. Additionally, a stiffness index, which can be interpreted easily, is used to evaluate the stiffness property of a PTPM. The stiffness index is able to relate the stiffness property to the direction of a wrench experienced by a PTPM in a task.

The remainder of this paper is organized as follows. The structures of PTPMs are described in Section 2. Section 3 presents the algebraic method using strain energy to analyze the stiffness of the PTPMs, followed by the analysis result comparison between