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Wrench-feasible workspace based optimization of the fixed and moving platforms for cable-driven parallel manipulators

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ABSTRACT

To improve the mechanical structure of cable-driven parallel manipulators (CDPMs), this paper optimizes the distribution of winches on the fixed platform and hinges on the moving platform by maximizing the total orientation wrench-feasible workspace (WFW). A convex analysis method is developed to estimate the total orientation WFW of CDPMs. The effect of the distribution of winches and hinges on the WFW is investigated for spatial 6-DOF redundantly restrained CDPMs, and the optimal size of the CDPMs can be obtained by the grouped coordinate descent (GCD) method. The optimization algorithm is implemented on a 6-DOF spatial CDPM with eight cables. The simulation results indicate that the mechanical property of CDPMs can be improved by optimizing the distribution of winches and hinges.

1. Introduction

Cable-driven parallel manipulators (CDPMs) are closed-loop mechanism, in which the fixed and moving platforms are connected by cables. The main components of CDPMs are fixed platform, moving platform, actuators, winches, hinges, and cables [1]. Using cables instead of rigid links makes CDPMs have several advantages over parallel manipulators with rigid links: (1) Long cables can be stored on reels easily, thus CDPMs provide large workspace potentially. (2) Cables are very light, and actuators can be installed on the ground, thus CDPMs have high payload-to-weight radio and they are appropriate to heavy-load and high-acceleration application. (3) As the special mechanism structure of CDPMs, they can be assembled/reassembled and reconfigured easily by modular design. Consequently, CDPMs have low manufacture cost and they are well-suited for many actual applications. A wide variety of CDPMs have been developed (e.g., [2–4]), and they have been applied in engineering equipments, such as aircraft wind tunnel test [5], large radio telescope [6], and so on.

Although CDPM has many desirable advantages, various factors may also limit its applications. Firstly, cables can only work in tension, i.e. cables are unable to push the moving platform. According to the tension property, several types of workspaces are defined, such as controllable workspace [7], wrench-closure workspace [8], and so on. Here the wrench-closure workspace (WCW) is defined as the set of poses at which any wrenches can be generated on the moving platform by all-positive cable tensions. Both the tension set and the required wrench set are unbounded. Thus WCW is only dependent on the pose of the moving platform, and it is an ideal workspace. Considering the cable tension range is limited in actual application, wrench-feasible condition is proposed for the tension in a prescribed range. Here wrench-feasible workspace (WFW) is a set of poses at which cables can balance any wrenches in a given set, and the tension in each cable remains within a prescribed range [9]. Finally, since CDPMs can be assembled/reassembled and reconfigured easily by modular design, the fixed and moving platforms can be adjusted and the workspace volume varies with the platform parameters. Therefore, the effects of the fixed and moving platforms on the WFW need to be investigated, and the platform parameters should be optimized. As the WFW satisfies the requirement of the engineering practice, it is applied as an optimization object while optimizing the CDPM in this paper.

As a special case of the WFW, many approaches have been proposed to determine the WCW [10–12]. For the general case of WFW, there are also a large number of research works. Bosscher et al. [13] used the available wrench set to analyze the boundary of the WFW for point-mass CDPMs. Then they extended their method to non-point-mass CDPMs [14], but the equations describing the WFW boundary are extremely complicated. Therefore, CDPMs are often considered at a known constant orientation, and it is difficult to obtain the total orientation WFW, in which all of the Euler angles in a given box should be feasible. Gouttefarde et al. [9] designed a numerical method to determine whether a box of poses of the moving platform is fully inside the WFW by interval analysis. But this method cannot ensure whether a pose is

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out of the WFW. Thus they extended this approach to determine whether a box of poses is fully out of the WFW [15]. However, the two conditions above for determining the WFW are only sufficient. Thus, it is not easy to obtain exact boundary of the WFW, and the algorithm for determining the WFW of spatial 6-DOF CDPMs is time-consuming. Bouchard et al. [16] proposed a hyperplane shifting method to get the hyperplane representation of the available wrench set. This method can quickly determine whether a pose of the moving platform is in the WFW or not.

Since the property of CDPMs varies with the mechanical structure, many methods have been proposed to design the optimal CDPM. Hay and Snyman [17] optimized the configuration of a planar CDPM by taking the dexterous workspace as performance index. Riechel and Ebert-Uphoff [18] used the WFW as performance index, and analyzed the effects of motor locations, tension bound and payload on the WFW, but they did not optimize the mechanical structure of CDPMs. Although the method proposed by Riechel and Ebert-Uphoff is suitable for non-point-mass CDPMs, the equations describing the WFW boundary are very complicated. Thus it is very difficult to compute the volume of the total orientation WFW, and it is not easy to expand their work to the 6-DOF CDPMs. Bosscher et al. [19] analyzed the constant orientation WFW for a cable-suspended robotic contour crafting system. They studied the distribution of cable tensions in different poses, but they also did not optimize the mechanical structure of CDPMs. Alikhani et al. [20] designed a large-scale CDPM, whose moving platform can only realize the translational motion. Thus the synthesis of this CDPM is based on the constant orientation WFW.

In general, an n-DOF CDPM with m cables can be classified into three kinds: the under-constrained mechanisms ($m<n+1$), completely constrained mechanisms ($m=n+1$) and redundantly constrained mechanisms ($m>n+1$) [21]. Since the number of cables m should be not less than $n+1$ for controlling an n-DOF CDPM, the equilibrium configuration of under-constrained CDPMs is feasible unless an external force is applied to the end-effector and the equilibrium is stable [22–23]. Pusey et al. [24] researched the workspace of an under-constrained 6-6 CDPM with different geometric configurations and sizes of the moving platform. Pusey defined the workspace was a set of points that the center of mass of the moving platform could be positioned when all the cables were in tensions and the moving platform was in equilibrium, but the tensions were not limited. Yao et al. [25] designed a completely constrained 4-3 CDPM, but the final parameters were not globally optimal. For the redundantly constrained mechanisms, the motion can be fully controlled, and the redundant cables can be employed to avoid the singular configurations. Azizian [26] applied the wrench-closure workspace as the object function to optimize a redundantly constrained CDPM, but the mechanical structure of the CDPM optimized by the dimensional synthesis is locally optimal and asymmetric.

Therefore, this paper investigates the total orientation WFW of spatial 6-DOF redundantly constrained CDPMs with different sizes of the fixed and moving platforms. Moreover, different distributions of winches on the fixed platform and hinges on the moving platform are studied for an already designed CDPM. In order to estimate the total orientation WFW, an algorithm is designed on a basis of convex analysis and hyperplane shifting method. The total orientation WFW is determined by checking whether each vertex of the orientation box is feasible. To maximize the total orientation WFW, the mechanical structure of CDPMs is optimized by the grouped coordinate descent (GCD) method. The GCD method can guarantee the final design is locally optimal and symmetric, and it is suitable for optimizing different types of CDPMs. The simulation experiments of optimizing CDPMs are implemented on a 6-DOF spatial CDPM with eight cables, and the effect of the distribution of hinges and winches on the WFW is verified. The simulation results indicate that the number of poses of the moving platform in the WFW obviously increase by optimizing the distribution of winches and hinges, and the mechanical property can be improved.

The paper is organized as follows. In section 2, the static model of an n-DOF CDPM is established. In section 3, the calculation method of the total orientation WFW is given for CDPMs. In section 4, the effects of the fixed and moving platforms on the WFW are investigated, and the optimization algorithm is designed. In section 5, simulation experiments are carried out on a 6-DOF CDPM with eight cables. Finally, several remarks are concluded.

2. Static modeling

The structural diagram of an n-DOF CDPM is shown in Fig. 1. Coordinate frame *Oxyz* is the base frame, and coordinate frame *Pxyz* is fixed on the moving platform. Here the gravity of cables is neglected, and each cable is seen as a straight line in the static balance.

Based on the force equilibrium of the moving platform, the static equation can be described as

$$\begin{cases} \sum\limits_{i=1}^{m} \mathbf{t}_i+\mathbf{F}_p=0 \\ \sum\limits_{i=1}^{m} \mathbf{r}_i\times\mathbf{t}_i+\mathbf{M}_p=0 \end{cases}, \tag{1}$$

where $\mathbf{t}_i$ is the cable tension, $\boldsymbol{P_iB_i}$ is the direction of $\mathbf{t}_i$, and $\mathbf{r}_i$ is vector $\boldsymbol{PP_i}$. Moreover, $\mathbf{F}_p$ and $\mathbf{M}_p$ are the external force and moment on the moving platform, respectively. Let $\mathbf{U}_i$ represent the unit vector of $\mathbf{t}_i$, and t_i represents the magnitude of the cable tension. Then the matrix form of Eq. (1) can be written as

$$\mathbf{AT}=\mathbf{W}, \tag{2}$$

where $\mathbf{T}=[t_1 \quad t_2 \quad \cdots \quad t_m]^T\in R^m$ is the cable tension vector, $\mathbf{W}=-\begin{bmatrix}\mathbf{F}_p\\ \mathbf{M}_P\end{bmatrix}\in R^n$ is the external wrench, and $\mathbf{A}=\begin{bmatrix}\mathbf{U}_1 & \mathbf{U}_2 & \dots & \mathbf{U}_m\\ \mathbf{r}_1\times\mathbf{U}_1 & \mathbf{r}_2\times\mathbf{U}_2 & \dots & \mathbf{r}_m\times\mathbf{U}_m\end{bmatrix}$ is the structure matrix. Moreover, $\mathbf{U}_i$ can be formulated by $\frac{\boldsymbol{OB}_i-\boldsymbol{OP}-\boldsymbol{PP}_i}{|\boldsymbol{OB}_i-\boldsymbol{OP}-\boldsymbol{PP}_i|}=\frac{\mathbf{b}_i-\mathbf{p}-\mathbf{R}\cdot\mathbf{r}_i^p}{|\mathbf{b}_i-\mathbf{p}-\mathbf{R}\cdot\mathbf{r}_i^p|}$, here the superscript p denotes the vectors in coordinate frame P, $\mathbf{R}$ represents the orientation of the coordinate frame P with respect to the coordinate frame O, and $\mathbf{R}$ can be formulated as

$$\mathbf{R}=\begin{bmatrix} \cos\alpha\cos\beta & -\sin\alpha\cos\gamma+\cos\alpha\sin\beta\sin\gamma & \sin\alpha\sin\gamma+\cos\alpha\sin\beta\cos\gamma \\ \sin\alpha\cos\beta & \cos\alpha\cos\gamma+\sin\alpha\sin\beta\sin\gamma & -\cos\alpha\sin\gamma+\sin\alpha\sin\beta\cos\gamma \\ -\sin\beta & \cos\beta\sin\gamma & \cos\beta\cos\gamma \end{bmatrix},$$

where α, β, and γ are the Z–Y–X Euler angles.

3. WFW analysis

The wrench-feasible condition is defined as cables can balance any external wrenches in a given set while each cable tension