Robotics and Computer-Integrated Manufacturing 30 (2014) 658–666

Contents lists available at ScienceDirect

Robotics and Computer-Integrated Manufacturing

journal homepage: www.elsevier.com/locate/rcim

Saturated proportional derivative control of flexible-joint manipulators

Ryan James Caverly [a,*], David Evan Zlotnik [a], Leila Jasmine Bridgeman [b], James Richard Forbes [a]

[a] Department of Aerospace Engineering, University of Michigan, FXB Building, 1320 Beal Ave., Ann Arbor, MI 48109-2140, USA
[b] Department of Mechanical Engineering, McGill University, 817 Sherbrooke St. West, Montreal, QC, Canada H3A 0C3

ARTICLE INFO

Article history:
Received 3 October 2013
Received in revised form
27 May 2014
Accepted 6 June 2014

Keywords:
Saturation avoidance
Flexible-joint manipulator
Actuator saturation
Proportional-derivative control

ABSTRACT

In this paper, the control of flexible-joint robotic manipulators while avoiding actuator saturation is investigated. Several proportional derivative controllers are developed, all of which disallow actuator saturation by guaranteeing that the applied torque is less than a specified maximum value. In particular, a Gibbs parameterization of the joint angles is included in the control laws, which allows for an increased control torque as compared to an Euler angle parameterization. An equilibrium point of the closed-loop system is proven to be asymptotically stable using the Lyapunov stability analysis. Moreover, the proposed control laws do not require any knowledge of the manipulator's mass, stiffness, or dissipation properties, and as such, are robust to modelling errors. The proposed controllers are tested on a single-link flexible-joint manipulator experimentally and on a two-link flexible-joint manipulator in simulation, and are compared to the performance of controllers found in the literature.

1. Introduction

Robotic manipulators are used extensively in many industries for repeatable tasks such as robotic welding and automated assembly. The joints of these manipulators will have a certain amount of flexibility because the gearing or belt drives that transmit torque from the actuators to the links have a finite stiffness [1]. In most cases this joint flexibility is left unmodelled and uncontrolled [1,2]. In order to avoid excitation of the natural frequencies of a manipulator's flexible joints, manipulator operators often reduce the acceleration of a maneuver and then wait for any vibrations to decay naturally [3]. In systems with limited natural damping this can take a significant amount of time, which is highly undesirable. Several authors have developed controllers for flexible-joint manipulators using widely varying techniques [4–10]. Some of these include passivity-based control [4,8], proportional-derivative (PD) control [5], and adaptive control [6,9].

A limiting factor in the control of flexible-joint robotic manipulators is actuator saturation. Actuators can only provide a finite amount torque to the manipulator being controlled, which can lead to performance limitations [11]. Unfortunately, powerful motors are generally large, heavy, and costly. Increasing the size of the motors, and hence the mass of the system, results in increased power requirements, as well as possible performance limitations [12]. For this reason, somewhat smaller or at least modestly sized motors are used in practice, resulting in restricted joint torques. As such, avoiding actuator saturation while simultaneously assuring asymptotic stability of the closed-loop equilibrium point is of great interest. Control laws that incorporate actuator saturation avoidance in the control of robotic manipulators have been considered in [13–16]. Hyperbolic tangent functions [13–16] are often used, as well as arctangent functions [14]. In the context of spacecraft attitude control, actuator saturation has been studied in [17–19]. In particular, the authors of [17] propose a PD control law that accounts for actuator saturation using a bounded saturation function.

The contribution of this paper is the formulation of two PD controllers based on the previous work of [17] for use on flexible-joint robotic manipulators. These controllers will be designed to disallow actuator saturation, while simultaneously guaranteeing asymptotic stability about a desired equilibrium point of the closed-loop system. Specifically, Gibbs parameters will be used in the control formulation, which will allow for greater proportional control further away from the desired set point compared to the use of Euler angles. Several authors have used a quaternion parametrization to describe the orientation of the end-effector [20–22], but to our knowledge the use of Gibbs parameters or quaternions has yet to be considered to parameterize the joint angles. As such, an additional contribution of this paper is the use of Gibbs parameters in the formulation of saturation controllers for robotic manipulators, which allows us to take advantage of the increased relative proportional control effort that comes from using Gibbs parameters instead of Euler angles. This increase in

* Corresponding author. Tel.: +1 734 764 3310; fax: +1 734 763 0578.
E-mail addresses: caverly@umich.edu (R.J. Caverly), dzlotnik@umich.edu (D.E. Zlotnik), leila.bridgeman@mail.mcgill.ca (L.J. Bridgeman), forbesrj@umich.edu (J.R. Forbes).

http://dx.doi.org/10.1016/j.rcim.2014.06.001
0736-5845/

relative control effort will be expanded upon in Section 3. The proposed controllers will be compared in simulation and experimental testing to controllers found in the literature. This paper will focus on planar robotic manipulators, since the torque needed to overcome gravitational forces is not explicitly accounted for. This paper builds upon the preliminary results presented in [23]. The present work is significantly different than that of [23], as more sophisticated manipulators are now considered, the closed-loop stability proofs have been generalized, and the PD control gains can be chosen independently for each manipulator joint, and additional PD gains have been included in the control laws to allow for further tuning of the controllers.

The remainder of this paper is as follows. In Section 2 the dynamic model of a multi-link robotic manipulator is presented. In Section 3 the proposed controllers are presented and shown to render the closed-loop system asymptotically stable, even in the presence of parameter uncertainty. In addition, the Gibbs parameter is introduced. An example involving a single-link manipulator is presented in Section 4. The proposed control laws are simplified for the single-link case, and the experimental results are presented. A numerical example involving a two-link manipulator is given in Section 5, which includes simulation results. Final remarks and possible future work are presented in Section 6.

2. System dynamics

Consider the multi-link flexible-joint manipulator shown in Fig. 1, whose equations of motion are [4–6,13,24]

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{D}\dot{\mathbf{q}}+\mathbf{K}\mathbf{q}=\hat{\mathbf{B}}\boldsymbol{\tau}_c+\mathbf{f}_{non}(\mathbf{q},\dot{\mathbf{q}}), \tag{1}$$

where $\mathbf{M}=\mathbf{M}^{\mathsf{T}}>0$ is the system's mass matrix, $\mathbf{K}=\mathbf{K}^{\mathrm{trans}}>0$ is the system's stiffness matrix, $\mathbf{D}=\mathbf{D}^{\mathsf{T}}\geq 0$ is the system's damping matrix, $\hat{\mathbf{B}}=[\mathbf{1}\ \mathbf{0}]^{\mathsf{T}}$ is the column vector that distributes the applied torques to the system, $\boldsymbol{\tau}_c=[\tau_{c,1}\ \tau_{c,2}\ \cdots\ \tau_{c,n}]^{\mathsf{T}}$ are the torques input to the system, $\mathbf{f}_{non}(\mathbf{q},\dot{\mathbf{q}})=-\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}$ is the column vector of nonlinear forces, $\mathbf{1}$ is the identity matrix of appropriate dimension, and $\mathbf{q}=[\theta_1\ \theta_2\ \cdots\ \theta_n\ \alpha_1\ \alpha_2\ \cdots\ \alpha_n]^{\mathsf{T}}$ are the generalized coordinates of the system. The angle $\theta_i,\ i=1,2,\ldots,n$ is the joint angle of the ith hub with respect to the previous link and the angle $\alpha_i,\ i=1,2,\ldots,n$ is the angle of the ith link with respect to the ith hub. Throughout this paper the subscript i will denote the ith hub and link. Gravitational forces have been neglected in this dynamic model, but could be accounted for by employing a feedforward control term as described in [4]. Note that the total energy of this system is $E=\frac{1}{2}\dot{\mathbf{q}}^{\mathsf{T}}\mathbf{M}\dot{\mathbf{q}}+\frac{1}{2}\mathbf{q}^{\mathsf{T}}\mathbf{K}\mathbf{q}$ and the time derivative of the total energy is $\dot{E}=-\dot{\mathbf{q}}^{\mathsf{T}}\mathbf{D}\dot{\mathbf{q}}+\dot{\mathbf{q}}^{\mathsf{T}}\hat{\mathbf{B}}\boldsymbol{\tau}_c$. Additionally, it can be shown that for serial manipulators, the matrix $(\dot{\mathbf{M}}-2\mathbf{C})$ is skew-symmetric [25].

In Fig. 1 $k_{s,i}$ is the stiffness of the ith flexible joint, J_{2i-1} is the second moment of mass of the ith hub, J_{2i} is the second moment of mass of the ith link, and m_p is the mass of the payload.

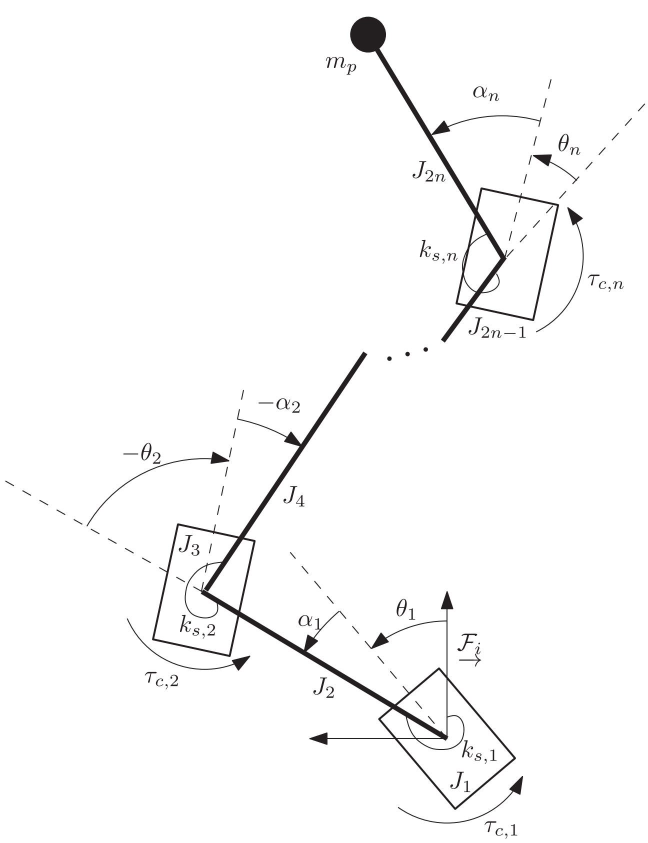

Fig. 1. Schematic of a multi-link flexible-joint robotic manipulator.

3. Control formulation

3.1. Gibbs parameterization

To motivate the structure of the proposed control law in the following section, the Gibbs parameter is introduced. Recall that the Gibbs parameter, $\mathbf{p}\in\mathbb{R}^3$, is related to the Euler axis/angle variables by [26]

$$\mathbf{p}=\mathbf{a}\ \tan(\theta/2), \tag{2}$$

where $\mathbf{a}$ is the Euler axis and θ is the Euler angle. The relationship between the angular velocity and the time rate of change of the Gibbs parameter is [26]

$$\dot{\mathbf{p}}=\tfrac{1}{2}(\mathbf{1}+\mathbf{p}^{\times}+\mathbf{p}\mathbf{p}^{\mathsf{T}})\boldsymbol{\omega}, \tag{3}$$

where $\boldsymbol{\omega}$ is the three-dimensional angular velocity of a body. Considering only rotation about a single axis, Eqs. (2) and (3) simplify to

$$p=\tan(\theta/2), \tag{4}$$

$$\dot{p}=\tfrac{1}{2}(1+p^2)\omega. \tag{5}$$

The joint angles of the flexible manipulator can be parameterized as $p_i=\tan(\theta_i/2),\ i=1,2,\ldots,n$ and collected as $\mathbf{p}=[p_1\ p_2\ \cdots\ p_n]^{\mathsf{T}}$. For the remaining sections of the paper, $\mathbf{p}$ will represent the column matrix composed of the scalar Gibbs parametrization of each individual flexible-joint angle, not a vector representing the three-dimensional Gibbs parameterization of a single joint. Similarly, $\boldsymbol{\omega}=\dot{\boldsymbol{\theta}}=[\dot{\theta}_1\ \dot{\theta}_2\ \cdots\ \dot{\theta}_n]^{\mathsf{T}}$ will represent the column vector composed of the scalar joint rates, and not a three-dimensional angular velocity. The vector containing the scalar joint angles is given by $\boldsymbol{\theta}=[\theta_1\ \theta_2\ \cdots\ \theta_n]^{\mathsf{T}}$. A plot illustrating Eq. (4) is given in Fig. 2. Notice that $|p|\geq|\theta/2|$ for all θ and $|p|\gg|\theta/2|$ as θ approaches 180°, where $\theta/2$ is used to match the linearization of p about $\theta=0°$. This property motivates the use of the Gibbs parameter instead of an Euler angle in a proportional control law, as additional control effort is demanded when the joint is further from the desired set point. The property shown in Eq. (5) will be useful in proving stability in Section 3.3.

3.2. Saturation avoidance control laws

Consider a PD control law of the following form:

$$\boldsymbol{\tau}_c=\mathbf{u}_p+\mathbf{u}_d, \tag{6}$$

where $\mathbf{u}_p=[u_{p,1}\ u_{p,2}\ \cdots\ u_{p,n}]^{\mathsf{T}}$ is the proportional control and $\mathbf{u}_d=[u_{d,1}\ u_{d,2}\ \cdots\ u_{d,n}]^{\mathsf{T}}$ is the derivative control. The control effort