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Constrained generalized Delaunay are plane spanners

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ABSTRACT

We look at generalized Delaunay graphs in the constrained setting by introducing line segments which the edges of the graph are not allowed to cross. Given an arbitrary convex shape C, a constrained Delaunay graph is constructed by adding an edge between two vertices p and q if and only if there exists a homothet of C with p and q on its boundary that does not contain any other vertices visible to p and q. We show that, regardless of the convex shape C used to construct the constrained Delaunay graph, there exists a constant t (that depends on C) such that it is a plane t-spanner of the visibility graph. Furthermore, we reduce the upper bound on the spanning ratio for the special case where the empty convex shape is an arbitrary rectangle to $\sqrt{2} \cdot (2l/s + 1)$, where l and s are the length of the long and short side of the rectangle.

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1. Introduction

A geometric graph G is a graph whose vertices are points in the plane and whose edges are line segments between pairs of vertices. Every edge in a geometric graph is weighted by the Euclidean distance between its endpoints. A graph G is called plane if no two edges intersect properly. The distance between two vertices u and v in G, denoted by $\delta_G(u, v)$, or simply $\delta(u, v)$ when G is clear from the context, is defined as the sum of the weights of the edges along a minimum-weight path between u and v in G. A subgraph H of G is a t-spanner of G (for $t \geq 1$) if for each pair of vertices u and v, $\delta_H(u, v) \leq t \cdot \delta_G(u, v)$. The smallest value t for which H is a t-spanner is the *spanning ratio* or *stretch factor* of H. The graph G is referred to as the *underlying graph* of H. The spanning properties of various geometric graphs have been studied extensively in the literature (see [1,2] for an overview of the topic).

Most of the research has focused on constructing spanners where the underlying graph is the complete Euclidean geometric graph. We study this problem in a more general setting with the introduction of line segment *constraints*. Specifically, let P be a set of points in the plane and let S be a set of line segments with endpoints in P, with no two line segments intersecting properly. The line segments of S are called *constraints*. Two points u and v can *see each other* or *are visible to each other* if and only if either the line segment uv does not properly intersect any constraint (i.e., does not intersect the interior of a constraint) or uv is itself a constraint. If two points u and v can see each other, the line segment uv is a *visibility edge*. The *visibility graph* of P with respect to a set of constraints S, denoted $Vis(P, S)$, has P as vertex set and all

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visibility edges as edge set. In other words, it is the complete graph on P minus all edges that properly intersect one or more constraints in S.

Visibility graphs have been studied extensively within the context of motion planning amid obstacles. Clarkson [3] was one of the first to study spanners in the presence of constraints and showed how to construct a linear-sized $(1+\epsilon)$-spanner of $Vis(P, S)$. Subsequently, Das [4] showed how to construct a spanner of $Vis(P, S)$ with constant spanning ratio and constant degree. Bose and Keil [5] showed that the Constrained Delaunay Triangulation is a $4\pi\sqrt{3}/9 \approx 2.42$-spanner of $Vis(P, S)$. The constrained Delaunay graph where the empty convex shape is an equilateral triangle was shown to be a 2-spanner of $Vis(P, S)$ [6]. We look at the constrained generalized Delaunay graph, where the empty convex shape can be any convex shape.

In the unconstrained setting, it is known that generalized Delaunay graphs are spanners [7], regardless of the convex shape used to construct them. A geometric graph G is a spanner when it satisfies the following properties (defined in Section 3.2): it is plane, it satisfies the α-diamond property, the spanning ratio of any one-sided path is at most κ, and it satisfies the visible-pair κ'-spanner property. In particular, G is a t-spanner for $t = 2\kappa\kappa' \cdot \max\left(\frac{3}{\sin(\alpha/2)}, \kappa\right)$. This upper bound is very general, but unfortunately not tight.

In special cases, better bounds are known. For example, when the empty convex shape is a circle, Dobkin et al. [8] showed that the spanning ratio is at most $\pi(1+\sqrt{5})/2 \approx 5.09$. Improving on this, Keil and Gutwin [9] reduced the spanning ratio to $4\pi/3\sqrt{3} \approx 2.42$. Recently, Xia showed that the spanning ratio is at most 1.998 [10]. We note that although Xia's proof is in the unconstrained setting, it still holds in the constrained setting. His proof is based on bounding the length of each edge on the path from a vertex s to t that does not intersect st with the arc of the empty circle defining the edge. The length of edges that cross st is then bounded in terms of the non-crossing edges. In the constrained setting, since the edges that do not cross st are still bounded by arcs of circles that are empty of visible points, his result holds.

Lower bounds are also studied for this problem. Bose et al. [11] showed a lower bound of 1.58, which is greater than $\pi/2$, which was conjectured to be the tight spanning ratio up to that point. Later, Xia and Zhang [12] improved this to 1.59.

Chew [13] showed that if an equilateral triangle is used instead, the spanning ratio is 2 and this ratio is tight. In the case of squares, Chew [14] showed that the spanning ratio is at most $\sqrt{10} \approx 3.16$. This was later improved by Bonichon et al. [15], who showed a tight spanning ratio of $\sqrt{4+2\sqrt{2}} \approx 2.61$.

In this paper, we show that the constrained generalized Delaunay graph G is a spanner whose spanning ratio depends solely on the properties of the empty convex shape C used to create it: We show that G satisfies the α_C-diamond property and the visible-pair κ_C-spanner property (defined in Section 3.2), which implies that it is a t-spanner of $Vis(P, S)$ for:

$$t = \begin{cases} 2\kappa_C \cdot \max\left(\frac{3}{\sin(\alpha_C/2)}, \kappa_C\right), & \text{if } G \text{ is a triangulation} \\ 2\kappa_C^2 \cdot \max\left(\frac{3}{\sin(\alpha_C/2)}, \kappa_C\right), & \text{otherwise.} \end{cases}$$

This proof is not a straightforward adaptation from the work by Bose et al. [7] due to the presence of constraints. For example, showing that a region contains no vertices that are visible to some specific vertex v requires more work than showing that this same region contains no vertices, since we allow vertices in the region that are not visible to v. Also, since the spanning ratio between some pairs of non-visible vertices of the constrained Delaunay graph may be unbounded (i.e., the length of the path between any two non-visible points can be made arbitrarily large by extending a constraint that blocks visibility), any proof of bounded spanning ratio needs to be restricted to the visible pairs of vertices. This implies that induction can only be applied to pairs of visible vertices, meaning that the inductive arguments cannot be applied in a straightforward manner as in the unconstrained case, since in the unconstrained case there is a spanning path between every pair of vertices.

Our spanning proof works directly on the Delaunay graph, instead of constructing the required paths using the Voronoi diagram as was done in [7]. This simplifies the algorithm for constructing these short paths, and also simplifies the proofs.

It is also worth noting that our definition of constrained Delaunay graph is slightly more general than the standard definition of these graphs: While it is usually assumed that all constraints are edges in the graphs, we do not require this and only add a constraint as an edge if it also satisfies the empty circle property used to construct the rest of the graph. Therefore, our result is slightly more general since we show that a subgraph of the standard constrained Delaunay graph is a plane spanner. We elaborate on this point in more detail in Section 2.

Finally, though the aforementioned result is very general, since it holds for arbitrary convex shapes, its implied spanning ratio is far from tight. To improve on this, in Section 4 we consider the special case where the empty convex shape C is a rectangle and show that it has spanning ratio at most $\sqrt{2} \cdot (2l/s + 1)$, where l and s are the length of the long and short side of C. This reduces the dependency on the aspect ratio from cubic (as implied by our general bound) to linear.

2. Preliminaries

Throughout this paper, we fix a bounded convex shape C. We assume without loss of generality that the origin lies in the interior of C. A *homothet* of C is obtained by scaling C with respect to the origin, followed by a translation. Thus, a homothet of C can be written as

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