Isothermal dissolution of small rising bubbles in a low viscosity liquid

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ABSTRACT

The isothermal dissolution of small single rising bubbles in a low viscosity liquid is numerically and experimentally studied. We propose a tracking-interface numerical method to analyze the dissolution of single bubbles rising in an isothermal liquid bath in the limit of small Weber numbers. The key elements of the method are the use of a frame of reference moving with the bubble and the application of different meshes to solve the mechanical and mass-diffusion problems. In addition, the gas concentration in the atmosphere over the bath, determined by mass balance of species in the gas column, is shown to be an essential component of the global dissolution problem in steady regime. Comparison with small oxygen bubbles rising in water has been carried out with remarkable agreement.

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1. Introduction

The evolution of gas bubbles in a liquid under buoyancy forces is a multifaceted multi-phase problem: it does not only involve the obvious fluid mechanical phenomena that can be mapped by the associated Reynolds and Weber numbers [1–3], size distribution, and number concentration [4], but also comprises the complex kinetics around the exchange of gases and vapors between the liquid and the bubble, and the heterogeneous distributions of such constituents in both liquid and gas domains [5–8]. There is a set of quite generic applications where gas exchange is, among the many different aspects that one may entertain in the problem of rising bubbles, the principal target factor to score performance. For example, in the petroleum industry, the number of wells demanding gas injection for oil recovery is soaring [9]. In this case, the high pressures and extreme conditions inside the well or oil bed promotes gas dissolution even at very low diffusivities [10]. On the other hand, CO_2 sequestration by injection in water reservoirs (excluding deep submarine injection in liquid phase), or the aeration of tanks or lakes demands a careful choice of bubble size as a function of injection depth to achieve optimal performance (total gas dissolution before the gas reaches the liquid surface) [11]. The same demand applies to micro algae cultivation tanks or bioreactors. To assess the dissolution process in realistic conditions, the Rising Bubble Apparatus (RBA) is a customary instrument [10,12]. In this device, used to measure what is called the minimum miscibility pressure, the evolution of the gas bubble is monitored by a video camera fixed to a rail parallel to the path of the rising bubble.

This work is motivated by the problem of gas injection into an aqueous environment (waste water, bioreactors, aquaculture, etc.) in the form of small bubbles. Many different techniques have been proposed for the production of microbubbles [13–21], with an ample understanding of the physical processes involved in their generation [21–23] and the operation parameters that govern their formation and final size. On the other hand, their transport in either closed channels [24] or in an open environment present radical differences not only in the obvious mechanical aspects, but most importantly – in the context of our work – in the mass exchange processes between the bubble and the environment. To fix ideas, we are interested in a reliable model to predict the evolution of a bubble of gas that slowly dissolves in an open environment, and whose size is sufficiently small to remain spherical – even subject to its buoyant rise through the surrounding liquid. In particular, our interest focuses on the total dissolution time and distance traveled by the evolving microbubble, with the idea to make that distance traveled as close as possible to the distance from the bubble source to the free surface of the liquid: in principle, this would allow an optimal dispersion of the gas throughout the liquid column, with minimal gas loses at the free liquid surface.

The paper is organized as follows. In Section 2, we propose a model made of a set of relatively general equations and boundary conditions that are resolved by an efficient pseudospectral technique. Then in Section 3 the experimental validation of our results is performed by the use of a setup inspired in the above mentioned Rising Bubble Apparatus without resorting to pressurization.

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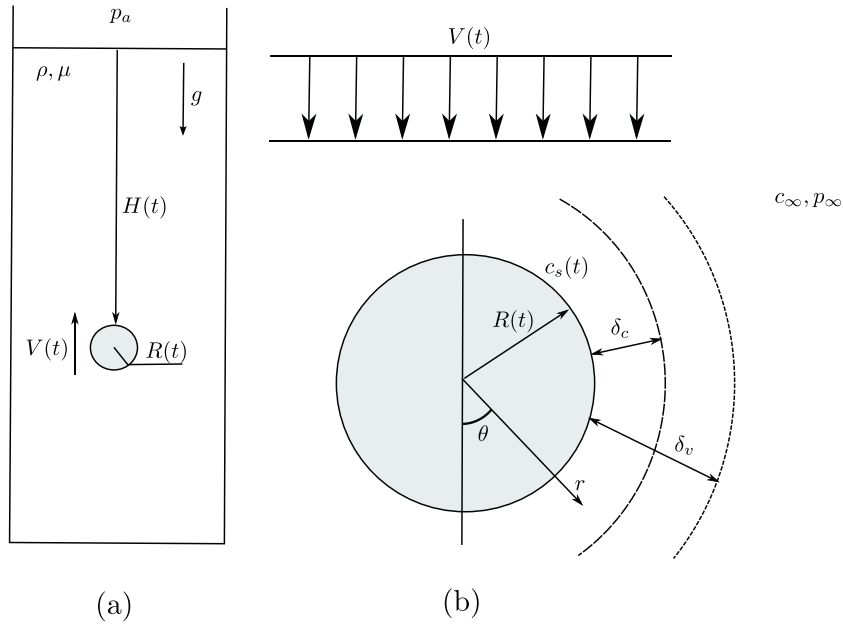 E-mail address: jmlopez@us.es (J.M. López-Herrera).

Fig. 1. Scheme of the problem.

Finally, Section 4 gathers results from both simulation and experiments to show a satisfactory agreement, thus validating our approach.

2. Problem formulation

Fig. 1 shows a sketch of the fluid configuration considered in this work. A spherical gas bubble with initial radius R_o is released in a still liquid bath of density ρ and viscosity μ from the bottom of a tube filled with liquid (water). The height of the liquid column is H_o and its upper free surface is open to air at atmospheric pressure p_a. The mass concentration of gas dissolved in the liquid bulk, c_∞, is assumed to be homogeneous. Denoting by χ the molar fraction of oxygen of the atmosphere at the upper free surface of the liquid, the partial pressure of O_2 in that location is given by Dalton's law, χp_a and, consequently, the concentration c_∞ is given by the Henry's law, $c_\infty = \chi p_a / K_H$. K_H is the Henry constant.

Due to the buoyancy force, the bubble rises at velocity $V(t)$. Simultaneously, in its way to the surface the bubble exchanges gas with the liquid, and its radius $R(t)$ varies due to both the change of liquid hydrostatic pressure and the dissolution of the gas in the surrounding liquid. We assume that the surface tension γ between the gas and the liquid is large enough compared to buoyancy and drag forces to ensure that the bubble remains spherical during its ascent, i.e. $\gamma \gg \rho g R_o^2$, where g is the acceleration of gravity. The bubble radius for which buoyancy and surface tension forces become comparable is $R_c = (\gamma/4\rho g)^{1/2}$. For an air bubble in water, that gives $R_c = 1.35$ mm. In this work, we will always consider sub-millimeter bubbles. Thus, the pressure inside the bubble, $p_g(t)$, is assumed nearly homogeneous, being its value given by the external liquid hydrostatic pressure and the surface tension force, $p_g(t) = p_a + \rho g H(t) + 2\gamma/R(t) - p_v(t) \equiv p_b(t) - p_v(t)$, where $p_v(t)$ is the liquid vapor pressure at the local liquid temperature, assuming that the kinetics of evaporation into the small bubble has characteristic times much smaller than the hydrodynamic times (i.e., assuming that the gas in the bubble is saturated with water vapor). $p_v(t)$ is given by:

$$p_v(t) = \left(\frac{p_a + \rho g H(t) + 2\gamma}{R(t)} \right) \exp\left[\int_{T_o}^{T} L_w(T) \Re^{-1} T^{-2} dT \right] \quad (1)$$

where $\Re$ is the specific gas constant, T_o the boiling temperature of the liquid at $p_g(t)$, and T its local temperature. L_w is the latent heat of vaporization. The specific systems onto which we will apply our model work at room temperature and thus, for the case of water, one has that the molar fraction of liquid vapor in the bubble is

$$\frac{p_v(t)}{(p_a + \rho g H(t) + 2\gamma/R(t))} \sim \exp[L_w \Re^{-1} T^{-2}(T - T_o)] \lesssim O(10^{-2}). \quad (2)$$

Assuming that the gas in the bubble has a dominant constituent, and that $p_a \gg \rho g H$, one may compare surface tension overpressure and vapor pressure through the non-dimensional number

$$\zeta_1 = \frac{p_a \exp[L_w \Re^{-1} T^{-2} \Delta T]}{\gamma/R_o} \quad (3)$$

where $\Delta T = T_o - T$. This number defines a critical bubble radius R_o^*, sufficiently below (above) which surface tension overpressure (vapor pressure) dominates. For water in atmospheric conditions, $R_o^* \simeq 0.05$ mm. Next, the relative weight of the hydrostatic overpressure and the vapor pressure of the liquid can be expressed by the non dimensional number

$$\zeta_2 = \frac{\rho g H_o}{p_a \exp[L_w \Re^{-1} T^{-2} \Delta T]} \quad (4)$$

The critical height sufficiently above (below) which hydrostatic pressure (vapor pressure) dominates is given by $H_o^* = p_a \exp[L_w \Re^{-1} T^{-2} \Delta T]/\rho g$. Our working conditions (air in water, atmospheric pressure) provide $H_o^* \simeq 160$ mm. Thus, both the liquid surface tension and vapor pressures can be neglected in the analysis for bubble sizes larger than R_o^*, and rising bubble columns sufficiently larger than H_o^*.

We use spherical coordinates (r, θ, φ) centered in the bubble and aligned with the vertical tube (see Fig. 1(b)) to analyze the problem. In this frame of reference the liquid far-velocity field corresponds to a uniformly descending velocity $V(t)$. Moreover, since we consider relatively small bubbles, in principle we assume dominance of viscous forces, although we will not dismiss inertia. Thus, the problem is axisymmetric as far as the Reynolds number $Re_o = 2(2\rho^2 g R_o^3/\mu^2)^{1/2}$ remains smaller than about 10^2 [25], and henceforth we drop the coordinate φ.

The incompressible axisymmetric Navier–Stokes equations for the liquid velocity $\mathbf{v}(r, \theta; t)$ and dynamic pressure $p_d(r, \theta; t)$ fields