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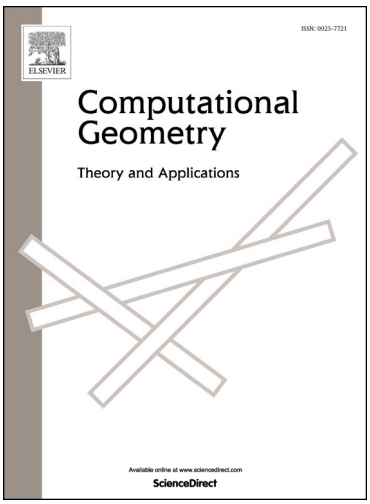


Cone-Based Spanners of Constant Degree

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# Cone-Based Spanners of Constant Degree[☆]

To the friendship and loving memory of Ferran, a passionate researcher and a great source of inspiration for me

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**Abstract**

We study spanning properties of sparse cone-based graphs of constant degree, parameterized by a positive integer $k > 1$. Cone-based graphs partition the space around each vertex into $k$ equiangular cones, and select at most one outgoing edge in each cone. This guarantees an out-degree of at most $k$, but the in-degree may be unbounded. To bound the in-degree, at most one incoming edge is retained in each cone. This yields a sparse graph of degree at most $2k$ (in-degree $k$ and out-degree $k$). In this paper we introduce the notion of *canonical $k$-cone graphs* and establish a spanning ratio of 16.76 for this class of graphs, provided that $k = 6k'$ and $k' \geq 5$. The spanning ratio decreases to 4.64 as $k'$ increases to 8. We show that both Yao-Yao and Theta-Theta graphs are canonical $k$-cone graphs and therefore they inherit these spanning ratios. Yao-Yao and Theta-Theta graphs differ only in the way edges are selected. Yao-Yao graphs select an edge of minimum Euclidean length, whereas Theta-Theta graphs select an edge of minimum orthogonal projection onto the cone bisector.

*Keywords:* cone graph, Yao, Theta, Yao-Yao, Theta-Theta, spanner

## 1. Introduction

Let $S$ be a set of $n$ points in the plane, and let $G$ be an undirected plane graph with vertex set $S$. The *length* of a path in $G$ is the sum of the Euclidean lengths of its constituent edges. The distance in $G$ between any two points $a, b \in S$ is the length of a shortest path between $a$ and $b$. We say that $G$ is a *spanner* if it preserves distances between each pair of points in $S$, up to a constant factor. Specifically, for a fixed integer $t \geq 1$, we say that $G$ is a *$t$-spanner* if any two points $a, b \in S$ at distance $|ab|$ in the plane are at distance

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