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Connecting a set of circles with minimum sum of radii

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Dedicated to our dear friend and colleague, Ferran Hurtado, who has inspired us to pose, to ponder, to question, to solve, to celebrate, and to enjoy life to the fullest. Thank you for inspiring this generation, and many generations to come!

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ABSTRACT

We consider the problem of assigning radii to a given set of points in the plane, such that the resulting set of disks is connected, and the sum of radii is minimized. We prove that the problem is NP-hard in planar weighted graphs if there are upper bounds on the radii and sketch a similar proof for planar point sets. For the case when there are no upper bounds on the radii, the complexity is open; we give a polynomial-time approximation scheme. We also give constant-factor approximation guarantees for solutions with a bounded number of disks; these results are supported by lower bounds, which are shown to be tight in some of the cases. Finally, we show that the problem is polynomially solvable if a connectivity tree is given, and we conclude with some experimental results.

1. Introduction

Problems of connectivity are among the most fundamental ones for many types of networks. Typically, these arise in a geometric setting, e.g., when considering the relation between the location of transmitters, the range of their transmissions, and their ability to connect. As a result, important aspects include the underlying geometry, the nature of connectivity, and the study of corresponding cost functions. Thus, connectivity problems bring together graph theory with computational geometry, a combination that was always dear to Ferran Hurtado; e.g., see [1] for a study of connectivity that mixes graphs and geometry.

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In this paper, we consider a natural connectivity problem, with a focus on the geometric aspects, arising from assigning ranges to a set of points, such that the resulting disk intersection graph is connected. More precisely, we are given a set of points $P = \{p_1, \ldots, p_n\}$ in the plane. Each point p_i is assigned a range r_i, inducing a disk of radius r_i. Two points p_i, p_j are adjacent in the connectivity graph H, if their disks intersect. The Connected Range Assignment Problem (CRA) requires an assignment of radii to P, such that the objective function $R = \sum_i r_i^\alpha$, $\alpha = 1$ is minimized, subject to the constraint that H is connected.

Problems of this type have been considered before and have natural motivations from fields including networks, robotics, and data analysis, where ranges have to be assigned to a set of devices, and the total cost is given by an objective function that considers the sum of the radii of disks to some exponent α. The cases $\alpha = 2$ or 3 correspond to minimizing the overall power. The motivation for the case $\alpha = 1$ arises from scanning the corresponding ranges with a minimum required angular resolution, so that the scan time for each disk corresponds to its perimeter, and thus radius.

1.1. Related work

There is a large body of literature on algorithmic methods for range assignment in wireless sensor and ad-hoc networks; see Calinescu et al. [6], Calinescu and Wan [7], Carmi and Katz [9], Carmi et al. [11], Caragiannis et al. [8], Lloyd et al. [20], Wan et al. [23] for a (small) selection of aspects and variants.

In the context of clustering, Doddi et al. [15], Charikar and Panigraphy [13], and Gibson et al. [17] consider the following problems. Given a set P of n points in a metric space with metric $d(i, j)$ and an integer k, partition P into a set of at most k clusters with minimum sum of either (a) cluster diameters, or (b) cluster radii. Thus, the most significant difference with our problem is the lack of a connectivity constraint. Doddi et al. [15] provide approximation results for (a). They present a polynomial-time algorithm, which returns $O(k)$ clusters that are $O(\log(\frac{n}{k}))$-approximate. For a fixed k, they transform an instance of their problem into a min-cost set-cover problem instance yielding a polynomial-time 2-approximation. They also show that the existence of a $(2 - \epsilon)$-approximation would imply $P = NP$. In addition, they prove that the problem in weighted graphs without triangle inequality cannot be efficiently approximated within any factor, unless $P = NP$. Note that every solution to (b) is a 2-approximation for (a). Thus, the approximation results can be applied to case (a) as well. A greedy logarithmic approximation and a primal-dual based constant factor approximation for minimum sum of cluster radii is provided by Charikar and Panigraphy [13].

Another often considered setting is the one in which the coverage of a given set of base stations is required. In this setting, Alt et al. [2] consider a closely related problem of selecting disk centers and radii such that a given set of points in the plane are covered by the disks. Like our work, they focus on minimizing an objective function based on $\sum_i r_i^\alpha$ and produce results specific to various values of α. The minimum sum of radii disk coverage problem (with $\alpha = 1$) is also considered by Lev-Tov and Peleg [18] in the context of radio networks. Again, connectivity is not a requirement. In a more geometric setting, Bilò et al. [5] provide approximation schemes for the minimum size k-clustering problem that requires dividing the set of centers into at most k clusters with minimum cluster cost.

A lot of work has also been done on radii/range assignment problems which require the special connectivity of "communication". The work of Clementi et al. [14] considers minimal assignments of transmission power to devices in a wireless network such that the network stays connected. In that context, the objective function typically considers an $\alpha > 1$ based on models of radio wave propagation. Furthermore, in the type of problem considered by Clementi et al. the connectivity graph is directed; i.e. the power assigned to a specific device affects its transmission range, but not its reception range. This is in contrast to our work in which we consider an undirected connectivity graph. See Fuchs [16] for a collection of hardness results of different (directed) communication graphs. Carmi et al. [10] prove that an Euclidean minimum spanning tree is a constant-factor approximation for a variety of problems including the *Minimum-Area Connected Disk Graph* problem, which equals our problem with the different objective of minimizing the *area* of the *union* of disks, while we consider minimizing the *sum* of the *radii* (or perimeters) of all disks. This can apply to a hybrid robot or sensor network system such as that described in Marinakis et al. [21]: Consider a situation in which a mobile robot is required to visit the region of each static network component, e.g., for environmental monitoring purposes, and requires constant one-way information from at least one static network component at all times; e.g., for navigational purposes. In this case, a reasonable objective is to reduce the area covered by the network while maintaining connectivity.

1.2. Our work

In this paper we investigate a variety of algorithmic aspects of the CRA problem. In Section 2, we show that for a given connectivity tree, an optimal solution can be computed efficiently. Section 3 gives a proof of NP-hardness for the problem when there is an upper bound on the radii, with full details for the case of planar weighted graphs and a sketch for planar point sets. Section 4 provides a number of approximation results for solutions with bounded number of disks. In Section 5, we present a PTAS for the case in which there are no upper bounds on the radii of the disks. These theoretical results are complemented by experimental results in Section 6. A concluding discussion with open problems is provided in Section 7.

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