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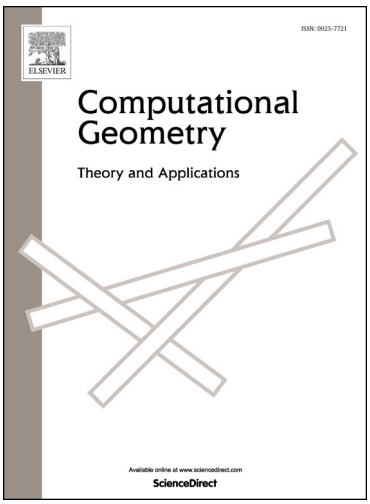

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Oswin Aichholzer, Ruy Fabila-Monroy, David Flores-Peñaloza, Thomas Hackl, Jorge Urrutia, Birgit Vogtenhuber

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Modem Illumination of Monotone Polygons*

Oswin Aichholzer[a,1], Ruy Fabila-Monroy[b,2], David Flores-Peñaloza[c,3], Thomas Hackl[a,4], Jorge Urrutia[d,5], Birgit Vogtenhuber[a,1,**]

[a] *Institute for Software Technology, Graz University of Technology, Graz, Austria*
[b] *Departamento de Matemáticas, Cinvestav, D.F. México, México*
[c] *Departamento de Matemáticas, Facultad de Ciencias, Universidad Nacional Autónoma de México, D.F. México, México*
[d] *Instituto de Matemáticas, Universidad Nacional Autónoma de México, D.F. México, México*

Abstract

We study a generalization of the classical problem of the illumination of polygons. Instead of modeling a light source we model a wireless device whose radio signal can penetrate a given number k of walls. We call these objects k-modems and study the minimum number of k-modems sufficient and sometimes necessary to illuminate monotone and monotone orthogonal polygons. We show that every monotone polygon with n vertices can be illuminated with $\lceil \frac{n-2}{2k+3} \rceil$ k-modems. In addition, we exhibit examples of monotone polygons requiring at least $\lceil \frac{n-2}{2k+3} \rceil$ k-modems to be illuminated.

For monotone orthogonal polygons with n vertices we show that for $k = 1$ and for even k, every such polygon can be illuminated with $\lceil \frac{n-2}{2k+4} \rceil$ k-modems, while for odd $k \geq 3$, $\lceil \frac{n-2}{2k+6} \rceil$ k-modems are always sufficient. Further, by presenting according examples of monotone orthogonal polygons, we show that both bounds are tight.

Keywords: Art Gallery Problem, Polygon Illumination, Modem Illumination, k-transmitter, k-Modem

1. Introduction

New technologies inspire new research problems, and wireless networking is a typical example of this. Nowadays, wireless technologies surround us everywhere. We use them in devices such as cellular phones, satellite communications, and, in our homes, we use wireless modems and routers to connect to the Internet.

This has triggered, among other things, the development of a new class of algorithms designed specifically to work with wireless networks, such as cellular networks, sensor networks, and *ad-hoc* networks [2, 20, 21]. The development of GPS, also a byproduct of wireless technologies, has allowed the development of so-called "local algorithms" for routing problems in cellular and *ad-hoc* networks [4, 14, 20] that allow relayed communication between any two nodes u and v of a network, at any time using only the position of u and v, as well as the current position of a message while traveling from u to v. For more details see [20, 21].

In this paper we study what we call the *Modem Illumination Problem*. This problem stems from our daily use of laptop computers and wireless modems. Experience shows that when trying to connect a laptop to a wireless modem (or router), there are two factors that have to be considered: the *distance* to the wireless modem and, perhaps most important in many buildings, the *number of walls* separating our laptop from the wireless modem. From now on, the

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**Corresponding author

Email addresses: oaich@ist.tugraz.at (Oswin Aichholzer), ruyfabila@math.cinvestav.edu.mx (Ruy Fabila-Monroy), dflorespenaloza@gmail.com (David Flores-Peñaloza), thackl@ist.tugraz.at (Thomas Hackl), urrutia@matem.unam.mx (Jorge Urrutia), bvogt@ist.tugraz.at (Birgit Vogtenhuber)

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