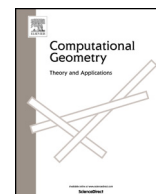




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Multiple covers with balls I: Inclusion–exclusion[☆]

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In memory of a good friend and trusted colleague

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ABSTRACT

Inclusion–exclusion is an effective method for computing the volume of a union of measurable sets. We extend it to multiple coverings, proving short inclusion–exclusion formulas for the subset of \mathbb{R}^n covered by at least k balls in a finite set. We implement two of the formulas in dimension $n = 3$ and report on results obtained with our software.

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1. Introduction

The work reported in this paper is motivated by configurations of balls that model the organization of DNA inside the nuclei of human cells: the *Spherical 1 Mega-base-pairs Chromatin Domain*, or *SCD model*, which is supported by high resolution microscopic observations [1,2]. It was recently confirmed that inside the chromosome territories in eukaryotic cells, DNA is compartmentalized in sequences of highly interacting segments of roughly the same length [3]. Each segment consists of about one million base pairs which are rolled up in a shape that resembles a round ball, and the shapes are tightly arranged within a restricted space.

Modeling such a configuration as a *packing* – in which the balls are rigid and allowed to touch but not overlap – is too restrictive because the rolled up base pairs push against each other and deform to cover more empty space than is otherwise possible. Similarly, modeling the configuration as a *covering* – in which the balls overlap and cover space without gaps – is not realistic because some empty space is necessary to facilitate the expression and replication of the DNA. We refer to [4] for a representative text in the rich mathematical literature on packings and coverings with balls. For the reason mentioned before, we are motivated to consider configurations that lie between these two extremes: the balls are allowed to overlap and they do not necessarily cover the entire space; see also [5]. Given such a configuration, we are interested in quantifications. For packings and coverings, it is customary to compute the *density*, which is the expected number of balls that contain a random point. This measure can also be used for more general configurations, but there are other choices. To mention one, we may be interested in the set of points each covered by exactly one ball; its volume is the difference between the volume of the union and of the 2-fold cover of the balls. It requires the ability to measure the set of points covered by at least two balls, which is a special case of the question addressed in this paper.

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Fig. 1. The first few non-zero rows of the Pascal triangle on the *left*, and of the alternating Pascal triangle on the *right*.

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