ARTICLE IN PRESS
COMGEO:1505

Computational Geometry ••• (••••) •••–•••

Contents lists available at ScienceDirect

Computational Geometry: Theory and Applications

www.elsevier.com/locate/comgeo

A note on interference in random networks ☆

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ARTICLE INFO

Article history:
Received 11 May 2016
Received in revised form 6 December 2016
Accepted 14 September 2017
Available online xxxx

Keywords:
Wireless network
Interference
Unit disk graph

ABSTRACT

The (maximum receiver-centric) interference of a geometric graph (von Rickenbach et al. 2005 [11]) is studied. It is shown that, with high probability, the following results hold for a set, V, of n points independently and uniformly distributed in the unit d-cube, for constant dimension d: (1) there exists a connected graph with vertex set V that has interference $O((\log n)^{1/3})$; (2) no connected graph with vertex set V has interference $o((\log n)^{1/4})$; and (3) the minimum spanning tree of V has interference $\Theta((\log n)^{1/2})$.

1. Introduction

Von Rickenbach et al. [11,12] introduce the notion of (maximum receiver-centric) interference in wireless networks and argue that topology-control algorithms for wireless networks should explicitly take this parameter into account. Indeed, they show that the minimum spanning tree, which seems a natural choice to reduce interference, can be very bad; there exists a set of node locations in which the minimum spanning tree of the nodes produces a network with maximum interference that is linear in the number, n, of nodes, but a more carefully chosen network has constant maximum interference, independent of n. These results are, however, *worst-case*; the set of node locations that achieve this are very carefully chosen. In particular, the ratio of the distance between the furthest and closest pair of nodes is exponential in the number of nodes.

The current paper continues the study of maximum interference, but in a model that is closer to a typical case. In particular, we consider what happens when the nodes are distributed uniformly, and independently, in the unit square. This distribution assumption can be used to approximately model the unorganized nature of ad-hoc networks and is commonly used in simulations of such networks [13]. Additionally, some types of sensor networks, especially with military applications, are specifically designed to be deployed by randomly placing (scattering) them in the deployment area. This distribution assumption models these applications very well.

Our results show that the maximum interference, in this case, is very far from the worst-case. In particular, for points independently and uniformly distributed in the unit square, the maximum interference of the minimum spanning tree grows only like the square root of the logarithm of the number of nodes. That is, the maximum interference is *not even logarithmic* in the number of nodes. Furthermore, a more carefully chosen network topology can reduce the maximum interference further still, to the cubed root of the logarithm of n.

☆ This work was partly funded by NSERC and CFI. A preliminary version of this paper appears in the *Proceedings of the 24th Canadian Conference on Computational Geometry (CCCG 2012)* [4].
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https://doi.org/10.1016/j.comgeo.2017.10.006
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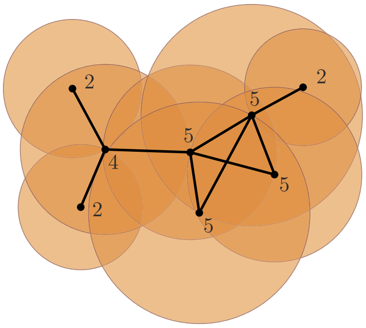

Fig. 1. A geometric graph G with $I(G) = 5$.

1.1. The model

Let $V = \{x_1, \ldots, x_n\}$ be a set of n points in $\mathbb{R}^d$ and let $G = (V, E)$ be a simple undirected graph with vertex set V. The graph G defines a set, $B(G)$, of closed balls $B_1, \ldots, B_n$, where B_i has center x_i and radius

$$r_i = \max\{\|x_i x_j\| : x_i x_j \in E\} \ .$$

(Here, and throughout, $\|xy\|$ denotes the Euclidean distance between points x and y.) In words, B_i is just large enough to enclose all of x_i's neighbors in G. The *(maximum receiver-centric) interference* at a point, x, is the number of these balls that contain x, i.e.,

$$I(x, G) = |\{B \in B(G) : x \in B\}| \ .$$

The *(maximum receiver-centric) interference* of G is the maximum interference at any vertex of G, i.e.,

$$I(G) = \max\{I(x, G) : x \in V\} \ .$$

Fig. 1 shows an example of a geometric graph G and the balls $B(G)$. Each node, x, is labeled with $I(x, G)$.

One of the goals of network design is to build, given V, a connected graph $G = (V, E)$ such that $I(G)$ is minimized. Thus, it is natural to consider interference as a property of the given point set, V, defined as

$$I(V) = \min\{I(G) : G = (V, E) \text{ is connected}\} \ .$$

A *minimum spanning tree* of V is a connected graph, $MST(V)$, of minimum total edge length. Minimum spanning trees are a natural choice for low-interference graphs. The purpose of the current paper is to prove the following results (here, and throughout, the phrase *with high probability* means with probability that approaches 1 as $n \to \infty$):

Theorem 1. *Let V be a set of n points independently and uniformly distributed in $[0, 1]^d$. With high probability,*

1. $I(MST(V)) \in O((\log n)^{1/2})$;
2. $I(V) \in O((\log n)^{1/3})$, *for* $d \in \{1, 2\}$; *and*
3. $I(V) \in O((\log n)^{1/3}(\log \log n)^{1/2})$, *for* $d \geq 3$.

Theorem 2. *Let V be a set of n points independently and uniformly distributed in $[0, 1]^d$. With high probability,*

1. $I(MST(V)) \in \Omega((\log n)^{1/2})$;
2. $I(V) \in \Omega((\log n)^{1/4})$.

1.2. Related work

This section surveys previous work on the problem of bounding the interference of worst-case and random point sets. A summary of the results described in this section is given in Table 1. In the statements of all results in this section, $|V| = n$.

The definition of interference used in this paper was introduced by von Rickenbach et al. [11] who proved upper and lower bounds on the interference of one dimensional point sets:

Theorem 4 *(von Rickenbach et al. 2005). For any $d \geq 1$, there exists $V \subset \mathbb{R}^d$ such that $I(V) \in \Omega(n^{1/2})$.*

The point set, V, in this lower-bound consists of any sequence of points $x_1, \ldots, x_n$, all on a line, such that $\|x_{i+1} x_i\| \leq (1/2)\|x_i x_{i-1}\|$, for all $i \in \{2, \ldots, n-1\}$. That is, the gaps between consecutive points decrease exponentially. This type of configuration is called an *exponential chain* and is also used in our lower bound construction. This lower bound is matched by an upper-bound:

Please cite this article in press as: L. Devroye, P. Morin, A note on interference in random networks, Comput. Geom. (2017), https://doi.org/10.1016/j.comgeo.2017.10.006