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Generalized confidence interval for the scale parameter of the power-law process with incomplete failure data

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ABSTRACT

System testing is very time-consuming and costly, especially for complex high-cost and high-reliability systems. For this reason, the numbers of failures needed for the developmental phase of system testing should be relatively small in general. To assess reliability growth of a repairable system, the generalized confidence interval for the scale parameter of the power-law process is studied concerning incomplete failure data. Specifically, some recorded failure times in the early developmental phase of system testing cannot be observed; this circumstance is essential to establish a warranty period or determine a maintenance phase for repairable systems. The simulation results show that the proposed generalized confidence interval is not a biased estimate even for small numbers of failures, and it provides the confidence intervals that have short average widths. Therefore, the proposed method is practically useful to save business costs and time during the developmental phase of system testing since only small numbers of failures are required, and it yields precise results. Additionally, the superiority of the proposed method is presented via a numerical study using two real examples.

1. Introduction

Most of the complex systems in use nowadays such as software systems, weapon systems, aircraft generators, and automobile engines are *repairable systems*. Before a repairable system is used or handed over to the customer, a vigorous system testing is usually conducted to explore all aspects of functionality and identify any inherent teething problems. Design changes are then made until the desirable reliability and dependability are attained.

The power-law process (PLP), which plays a crucial role in this research, is the most widely used model to study and assess reliability growth of repairable systems. Over the past few decades, much has been studied regarding the estimation and hypothesis testing concerning parameters of PLP. However, most research is developed based on complete failure data when all failure times are concisely and accurately recorded (Verma and Kapur, 2005), as shown in Fig. 1.

In practical situations, incomplete failure data are a common and expected occurrence during the developmental phase of system testing. Various types of incompleteness can and do occur in the aspect of missing positions for the failure data. One of the most frequently encountered scenarios concerns missing position located in the early developmental phase of system testing. This type of missing pattern can be caused by many reasons. For example, a new data-recording engineer may not have the expertise to determine the exact failure times during the early stage of the development process due to lack of experience.

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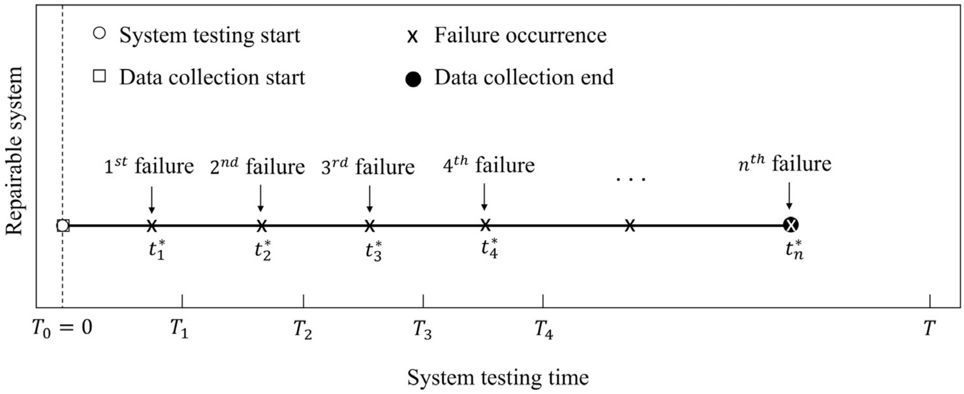

Fig. 1. Complete failure data with a predetermined number of failures (n).

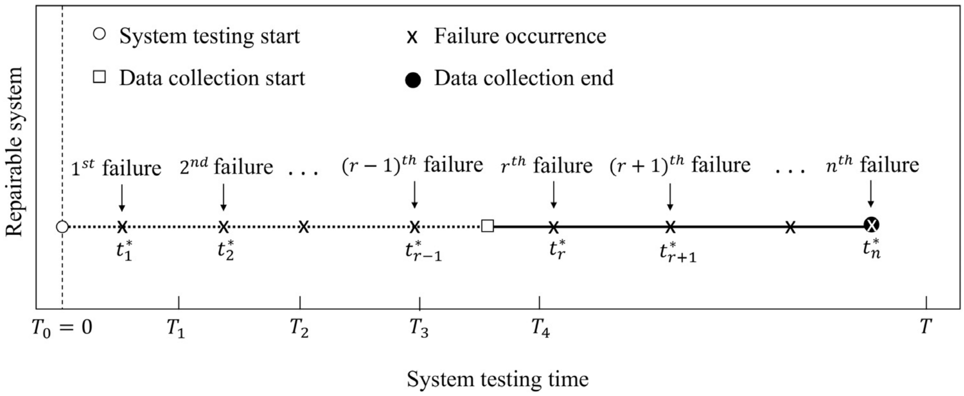

Fig. 2. Incomplete failure data with a predetermined number of failures (n).

Recently, there has been growing interest concerning the particular case of incomplete failure data when missing failure times only occur in the early developmental phase of system testing; it is essential to establish a warranty period or determine a maintenance phase for repairable systems. Yu et al. (2008) developed classical methods for statistical inferences and prediction analyses for the PLP concerning the first $r-1$ failure times is missing, while Tian et al. (2011) considered a Bayesian estimation and prediction for the PLP in the presence of left-truncated data. Latest, Na and Chang (2017) suggested a method to estimate the failure intensity function of the PLP using missing data at the initial recording stage from multiple machines for applications in the field of tank maintenance. In this paper, we study a generalized confidence interval for the scale parameter of the PLP with incomplete failure data when missing failure times occur in the early developmental phase of system testing. Using the notations given by Yu et al. (2008), we assume that $t_1, t_2, \ldots, t_{r-1}$ $(1 \leq r < n)$ are missing when the number of failures (n) is predetermined. That is, only failure times $t_r, t_{r+1}, \ldots, t_n$ can be observed, as shown in Fig. 2.

This study aims to compare the proposed generalized confidence interval and the classical confidence interval and determine which method is better to assess the system reliability during the developmental phase when numbers of failure are small and some recorded failure times in the early developmental phase cannot be observed.

2. Maximum likelihood estimates with missing data

The concept of maximum likelihood estimation of parameters with missing data was first proposed by Dempster et al. in 1977 (Dempster et al., 1977). Complete observations $g(Y_{obs}, Y_{miss}|\gamma, \beta)$ are related to the missing data specification $f(Y_{obs}|\gamma, \beta)$ by

$$f(Y_{obs}|\gamma, \beta) = \int g(Y_{obs}, Y_{miss}|\gamma, \beta) dY_{miss}. \tag{1}$$

Here, we assume that the failure process follows the PLP with intensity function

$$\lambda(t) = \gamma \beta t^{\beta - 1}, \tag{2}$$