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Inference and Mixture Modeling with the Elliptical Gamma Distribution

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Abstract

The authors study modeling and inference with the *Elliptical Gamma Distribution* (EGD). In particular, Maximum likelihood (ML) estimation for EGD scatter matrices is considered, a task for which the authors present new fixed-point algorithms. The algorithms are shown to be efficient and convergent to global optima despite non-convexity. Moreover, they turn out to be much faster than both a well-known iterative algorithm of Kent & Tyler and sophisticated manifold optimization algorithms. Subsequently, the ML algorithms are invoked as subroutines for estimating parameters of a mixture of EGDs. The performance of the methods is illustrated on the task of modeling natural image statistics—the proposed EGD mixture model yields the most parsimonious model among several competing approaches.

Key words: Maximum Likelihood; Elliptical Gamma; Elliptically Contoured Distributions; Conic Geometric Optimization; Nonconvex global optimization

1. Introduction

Non-Gaussian distributions occur in a multitude of applications. They may capture manifold structure (Banerjee et al., 2005; Pennec, 2006; Chikuse, 2003), or elicit sparsity (Kotz et al., 2001; Seeger and Nickisch, 2011), express heavy or light tailed behavior (Ollila et al., 2012; Kotz et al., 2001), characterize independence (Lee et al., 1999; Hyvärinen, 1999), or help us model a variety of other properties of data.

We focus on a particular non-Gaussian distribution: the *Elliptical Gamma (EG) Distribution* (EGD) (Kotz, 1975; Koutras, 1986). The (mean-zero) EG density (when it exists) for a point $\boldsymbol{x} \in \mathbb{R}^q$ is given by

$$p_{\text{eg}}(\boldsymbol{x}; \boldsymbol{\Sigma}, a, b) := \frac{\Gamma(q/2)}{\pi^{q/2}\Gamma(a) b^a |\boldsymbol{\Sigma}|^{1/2}} (\boldsymbol{x}^\top \boldsymbol{\Sigma}^{-1} \boldsymbol{x})^{a-q/2} \exp(-b^{-1} \boldsymbol{x}^\top \boldsymbol{\Sigma}^{-1} \boldsymbol{x}), \quad (1.1)$$

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