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Spectral approach to parameter-free unit root testing ☆

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ABSTRACT

A relatively simple frequency-type testing procedure for unit root potentially contaminated by an additive stationary noise is introduced, which encompasses general settings and allows for linear trends. The proposed test for unit root versus stationarity is based on a finite number of periodograms computed at low Fourier frequencies. It is not sensitive to the selection of tuning parameters defining the range of frequencies so long as they are in the vicinity of zero. The test does not require augmentation, has parameter-free non-standard asymptotic distribution and is correctly sized. The consistency rate under the alternative of stationarity reveals the relation between the power of the test and the long-run variance of the process. The finite sample performance of the test is explored in a Monte Carlo simulation study, and its empirical application suggests rejection of the unit root hypothesis for some of the Nelson–Plosser time series.

1. Introduction

A common starting point in time series analysis is the assessment of whether a stationary or non-stationary type of statistical/econometric model best characterizes the properties of the data under investigation. This has strong methodological and statistical implications. There exists a broad and still evolving literature aimed at determining the presence of a unit root in economic data sets.

One of the most widely employed unit root testing procedures is the classical Dickey Fuller (DF) test (Dickey and Fuller, 1979), and its augmented (ADF) version (Said and Dickey, 1984). The DF method has undergone numerous refinements, enabling its deep theoretical understanding and practical use. The most prominent ones include Phillips (1987), Phillips and Perron (1988), Elliot et al. (1996) and Ng and Perron (2001) while useful surveys on issues associated with unit root testing can be found in Stock (1994), Maddala and Kim (1998) and Phillips and Xiao (1998). For more recent developments on unit root testing see e.g. Westerlund (2014) and Shelef (2016).

At the core of these tests lie assumptions about the structural form of the times series studied, say $y_j, j = 1, \ldots, n$. For example, the simplest form of the DF test examines the null hypothesis that $\{y_j\}$ has pure unit root against an AR(1) stationary alternative. Other settings allow for inclusion of an intercept or intercept and time trend which yield different, complex asymptotic distributional representations, and hence altered critical values (MacKinnon, 1991). In turn, the Augmented Dickey Fuller (ADF) test centers on the null hypothesis of an ARIMA(p, 1, 0) process against the stationary ARMA($p + 1$, 0, q) alternative, see e.g. Cheung and Lai (1995) or Lopez (1997). Augmentation or selection of the appropriate

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lag order p is needed to absorb the additional dependence structure as well as computing adjusted critical values, see e.g. Cheung and Lai (1995), Ng and Perron (1995), Elliot et al. (1996), Ng and Perron (2001), Perron and Qu (2007) where these issues are studied thoroughly. Breitung (2002), on the other hand, suggests a different type of unit root test which is based on the variance-ratio statistic. The key advantage of this test is that it does not require specification of the short-run dynamics (augmentation).

Alternative approaches that focus on testing for stationarity versus unit root are inherently adaptable to a wide range of dependence structures of an underlying stationary time series (residuals), from short to long or negative memory. The most prominent of these is the KPSS test by Kwiatkowski et al. (1992), and its subsequent developments, see e.g. Giraitis et al. (2006). The non-standard asymptotic distributions of the corresponding test statistics are again complex and rather intractable, and require estimation of the long-run variance of the associated series. The need for augmentation arises again, this time in the context of estimation of the long-run variance, which may complicate the practical implementation of these tests.

This paper suggests a new and relatively simple frequency-type method for testing for a unit root (potentially contaminated by an additive stationary noise) versus stationarity, which makes use of fundamental properties of the spectrum and periodogram in the vicinity of zero frequency. More precisely, under unit root the periodogram has a sharp peak at zero frequency, and therefore testing can be based on a finite number of periodograms computed at low Fourier frequencies, $u_l, \ldots, u_k;\ u_p, \ldots, u_q$. Theory points out the need for k, q to be small, but it does not require data based selection of tuning parameters l, k, p, q, different values of which yield similar size/power performance of the test. Hence, the range of frequencies can be selected a priori. Furthermore, the frequency-type method allows testing for a unit root contaminated by an additive stationary noise, which constitutes the main structural novelty of the paper. The method is easily implemented and does not require augmentation. Under the null it has a parameter-free, tractable asymptotic distribution with critical values that do not require finite sample adjustment and yield correct size for sample sizes $n = 64, 128, 256, 1024$. Monte Carlo simulation results show that the test is well-sized and has satisfactory power under different data specifications.

The rest of the paper is organized as follows. In Section 2 we introduce the low-frequency-type testing procedure for a unit root (Q test) and derive its theoretical properties. In its current format, the differenced unit root ∇x_t is required to be a stationary linear process but an extension to a non-linear framework can be considered as well. The consistency rate under the alternative of stationarity reveals the relation between the power of the test and the long-run variance of ∇x_t. In Section 3 we analyze the finite sample properties of the Q test for a number of data generating models. Of particular interest is the case of a pure unit root augmented by an additive stationary noise, where we compare performance the Q test with the ADF and Breitung tests. Finally, Section 4 contains the empirical application of the Q test to the popular set of time series studied in Nelson and Plosser (1982) and Schotman and van Dijk (1991). Results indicate that for some time series the null hypothesis of unit root can be rejected. Proofs of the main results are contained in the Appendix.

2. Low-frequency-periodogram-type test

In this section we present a new frequency domain procedure for testing for a unit root in a time series potentially contaminated by an additive stationary noise. The idea behind such an approach is based on the observation that the periodogram ('spectrum') of a unit root process has high-order singularity at zero frequency. Construction of the test takes into account the fundamental asymptotic properties of a vector of the periodograms $\big(I(u_1), \ldots, I(u_q)\big)$ and the discrete Fourier transforms (DFT) computed at low Fourier frequencies $u_1, \ldots, u_q$ for a fixed q. The main advantages of frequency-type methods are well documented in Choi and Phillips (1993): (i) no explicit structural form of the error terms is required, (ii) the resulting limiting distributions are parameter-free, (iii) a strong peak of the periodogram at zero frequency under unit root is taken into account, (iv) such tests are predominantly correctly sized. Our objective in this section is to devise a test with parameter-free asymptotic distribution, the tuning parameters of which do not require data based selection. Compared to spectral-type testing procedures by Choi and Phillips (1993) or Fan and Gençay (2010), this is a very different type of test with a different limit distribution. It does not require estimation of the spectral density, uses a preselected finite number of Fourier frequencies and is easy to compute.

To proceed with the definition of the test and its theoretical properties, we set up the null and alternative hypotheses. A process $\{\xi_j, j \in \mathbb{Z}\}$ is said to be a short memory process if it has absolutely summable autocovariances $\gamma_\xi(k) = \mathrm{Cov}(\xi_k, \xi_0)$,

$$\sum_{k=-\infty}^{\infty} |\gamma_\xi(k)| < \infty, \qquad \sum_{k=-\infty}^{\infty} \gamma_\xi(k) > 0. \tag{1}$$

We describe first the hypotheses of unit root with no trend.

Hypothesis $\boldsymbol{H}_0$ *(unit root).* We say that the random variables $y_j, j = 1, \ldots, n$ satisfy the null hypothesis H_0 of unit root with an additive noise if

$$y_j = x_j + \varepsilon_j, \quad j \geq 1, \text{ where } x_j = x_{j-1} + \xi_j, \tag{2}$$

and $\{\xi_j\}$ and $\{\varepsilon_j\}$ are zero mean short memory processes, as defined above in (1).

The alternative hypothesis to H_0 includes a stationary process with unknown mean.