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Semiparametric score driven volatility models

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ABSTRACT

A new semiparametric observation-driven volatility model is proposed. In contrast to the standard semiparametric generalized autoregressive conditional heteroskedasticity (GARCH) model, the form of the error density has a direct influence on both the semiparametric likelihood and the volatility dynamics. The estimator is shown to consistently estimate the conditional pseudo true parameters of the model. Simulation-based evidence and an empirical application to stock return data confirm that the new statistical model realizes substantial improvements compared to GARCH type models and quasi-maximum likelihood estimation if errors are fat-tailed and possibly skewed.

1. Introduction

Models for time varying volatility are very popular in statistics and economics as parsimonious descriptions of a range of empirical stylized facts. Starting with the seminal (G)ARCH papers by Engle (1982) and Bollerslev (1986), many variations and extensions of the GARCH class of models have been proposed in the literature, such as the GARCH-M model of Engle et al. (1987), the EGARCH model of Nelson (1991), the GJR-GARCH model of Glosten et al. (1993), and the Threshold GARCH (TGARCH) model by Zakoian (1994). For a recent textbook treatment of GARCH models, see for example Francq and Zakoïan (2010). Interestingly, Quasi Maximum Likelihood Estimation (QMLE) of GARCH models based on the (possibly incorrect) assumption of conditional normality for the error terms still yields $\sqrt{T}$-consistent estimators in many cases, with T denoting the sample size; see Weiss (1986), Lee and Hansen (1994), and Lumsdaine (1996). This is important, as the conditional normality assumption is often rejected by empirical data. The statistical efficiency loss, however, can be considerable. Therefore, many empirical studies assume non-Gaussian conditional distributions, such as for example a (skewed) Student's t distribution, see Baillie and Bollerslev (1989) and Bauwens and Laurent (2005).

Semiparametric GARCH models have been proposed to avoid the potential efficiency loss and bias problems caused by assuming an incorrect parametric family of conditional distributions. Engle and Gonzalez-Rivera (1991) show that more efficient parameter estimates can be obtained by estimating the error density nonparametrically, though some efficiency loss remains. Drost and Klaassen (1997) and Sun and Stengos (2006) develop kernel-based estimators and establish the semiparametric efficiency bounds for parameter estimation. They characterize the conditions under which one can adaptively estimate a subset of the model's parameters, i.e., one can achieve the same asymptotic efficiency as if the true error density were known.

Our main contribution is to provide a new semiparametric model for time-varying volatility in which the form of the error distribution is directly linked to the volatility dynamics using the generalized autoregressive score (GAS) dynamics of Creal et al. (2011, 2013) and Harvey (2013). By doing so, we extend the class of GAS models to the semiparametric context.

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So far, all GAS applications have been fully parametric. We find that if the true data generating process is conditionally fat-tailed and skewed, the semiparametric model regains a substantial part of the efficiency loss due to the use of an incorrectly assumed Gaussian or Student's t distribution.

In a fully parametric context, the GAS framework of Creal et al. (2013) has been applied successfully to a range of empirical applications, including dynamic location models, dynamic correlation and copula and dynamic count data models; see for example Creal et al. (2011), Oh and Patton (2013), Harvey and Luati (2014), Lucas et al. (2014), Andres (2014), and Creal et al. (2014). Parameter estimation for GAS models is straightforward, since the likelihood function can be specified in closed form. The asymptotic properties of GAS processes and of the maximum likelihood estimator for GAS models were studied in Blasques et al. (2014a,b). Our current model estimates the conditional density of the innovations non-parametrically, and uses the estimate to construct both the semiparametric maximum likelihood estimator, as well as the dynamics of the time varying volatility using the score of the density estimate. The information theoretic optimality properties of the score of the conditional density as a driver for the time varying parameters were shown in Blasques et al. (forthcoming).

Parameter estimation for our model is straightforward and carried out in two steps. First, we estimate the parameters by assuming a normal or Student's t conditional quasi density for the error terms. The assumed quasi density also influences the volatility dynamics through the assumed GAS dynamics. From this we obtain the standardized residuals and the kernel density estimate of the error density. Second, the kernel density estimate is used to re-estimate the model's static parameters, resulting in the Semiparametric Maximum Likelihood Estimator. Again, the nonparametric estimate of the density provides new dynamics for the dynamic volatility parameter. This process can be iterated further until the semiparametric likelihood estimate does not increase further. We prove the theoretical properties of this approach. It is useful at this stage to note that the semiparametric nature of our model relates to the nonparametric estimation of the conditional error density as in Drost and Klaassen (1997) and Sun and Stengos (2006), and not to the nonparametric estimation of the volatility dynamics as in Pagan and Schwert (1990), Linton and Mammen (2005) and Yang (2006). The latter would be interesting in its own right, but is not pursued here.

The remainder of this paper is organized as follows. Section 2 introduces the model. Section 3 lays down the basic theoretical conditions for maximum likelihood estimation of this model. Section 4 provides Monte Carlo evidence of the new model's performance. Section 5 presents an empirical application to daily euro–dollar exchange rate data. Section 6 concludes.

2. The semiparametric volatility model

Consider the GARCH(1, 1) model

$$y_t = \mu + \xi_t = \mu + \tilde{f}_t^{1/2}\varepsilon_t, \quad \varepsilon_t \sim q(\varepsilon_t), \tag{1}$$

$$\tilde{f}_{t+1} = \tilde{\omega}\cdot(1-\tilde{\alpha}-\tilde{\beta}) + \tilde{\alpha}\xi_t^2 + \tilde{\beta}\tilde{f}_t, \tag{2}$$

with $y_t \in \mathbb{R},\ t = 1,\ldots,T$, a univariate time series with conditional mean $\mu \in \mathbb{R}$, $q(\cdot)$ the density of the standardized error term ε_t, $\tilde{f}_t \in \mathbb{R}^+$ the conditional variance, and $\tilde{\omega}, \tilde{\alpha}, \tilde{\beta} > 0$ static parameters satisfying $\tilde{\alpha} + \tilde{\beta} < 1$. The standardized density $q(\cdot)$ does not depend on $\tilde{f}_t$. Note that we can easily replace the conditional mean μ by a non-constant conditional mean μ_t involving exogenous regressors or autoregressive moving average components. Moreover, we can include lags of ξ_t and $\tilde{f}_t$ in (2).

If the distribution $q(\cdot)$ of ε_t is unknown, one can still proceed by assuming the normal distribution as a quasi density and obtain consistent estimates, see Francq and Zakoïan (2010). As mentioned in the introduction, however, the efficiency loss of this approach can be substantial. If $q(\cdot)$ itself needs to be estimated (nonparametrically), model (1)–(2) is called a semiparametric GARCH(1, 1) model. Estimation is usually performed in two steps. First, one estimates the static parameters $(\mu, \tilde{\omega}, \tilde{\alpha}, \tilde{\beta})$ using a Quasi Maximum Likelihood estimator (QMLE) based on the standard normal distribution. Using the standardized residuals $\hat{\tilde{f}}_t^{-1/2}\cdot(y_t - \hat{\mu})$ and standard nonparametric density estimation techniques, one then obtains an estimate $\hat{q}(\cdot)$ of the standardized error density. The estimated density is used to estimate the parameters $(\mu, \tilde{\omega}, \tilde{\alpha}, \tilde{\beta})$ in a second step by semiparametric ML, assuming the estimated density $\hat{q}(\cdot)$ is the true density. Engle and Gonzalez-Rivera (1991) use the discrete maximum penalized likelihood estimator (DMPLE) of Tapia and Thompson (1978) for density estimation, followed by a BHHH algorithm to maximize the semiparametric likelihood. Drost and Klaassen (1997) and Sun and Stengos (2006) use standard kernel density methods to estimate the error density and develop a Newton–Raphson algorithm to maximize the semiparametric likelihood. Their estimator achieves the same asymptotic efficiency for $(\tilde{\alpha}, \tilde{\beta})$ as the maximum likelihood estimator (MLE).

Our new semiparametric GAS(1, 1) model is given by

$$y_t = \mu + \xi_t = \mu + \exp(f_t/2)\varepsilon_t, \quad \varepsilon_t \sim q(\varepsilon_t), \tag{1'}$$

$$f_{t+1} = \omega\cdot(1-\beta) + \alpha s_t + \beta f_t, \tag{2'}$$

$$s_t = \frac{\partial \ln p(y_t|f_t)}{\partial f_t} = -\frac{1}{2}\left(e^{-f_t/2}\xi_t\cdot\nabla q(e^{-f_t/2}\xi_t) - 1\right), \tag{3'}$$