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Managing risk with a realized copula parameter

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ABSTRACT

A dynamic copula model is introduced, in which the copula structure is inferred from the realized covariance matrix estimated from within-day high-frequency data. The estimation is carried out in a method-of-moments fashion using Hoeffding's lemma. Applying this procedure day by day gives rise to a time series of daily copula parameters which can be approximated by an autoregressive time series model. This allows one to capture time-varying dependence. In an application to portfolio risk-management, it is found that this time-varying realized copula model exhibits very good forecasting properties for the one-day ahead value at risk.

1. Introduction

Realized variance (RV) and realized covariance (RCov) estimated from high-frequency intraday data have proved to be accurate ex-post measures for conditional variance and conditional covariance of daily returns. Nonparametric in nature, RV and RCov permit the econometrician to obtain proxies for financial (co)volatility without having to specify a priori an explicit and potentially misspecified volatility model (Andersen et al., 2001a,b). This insight has spurred intensive research in the field and has led to widespread use of measures of RV and RCov in numerous applications in finance, such as asset pricing, portfolio optimization, risk management, and volatility forecasting.

The present article continues this agenda. We estimate RCov matrices from high-frequency intraday data and take them as ex-post proxies for daily conditional covariance. We complement these estimates by making assumptions on the marginal distributions of daily returns and the copula associated with their joint multivariate distribution. Based on these assumptions, we estimate the copula shape parameters by means of a covariance moment condition provided by Hoeffding's lemma. The procedure yields estimates of daily copula shape parameters as materialized in RCov. The resulting time series of RCov-implied copula shape parameters is subsequently modeled by standard time series techniques, thereby allowing the dependence structure to be time-varying with the business cycle. We therefore call our approach the *realized copula (RCop)* model.

For risk-management purposes at a daily frequency, the benefits of using copulae to capture salient features of multivariate dependence are widely recognized; see Jin (2010) and references therein. Yet, purely RV-based models often work with a conditional multivariate Gaussian structure. The RCop model allows to drop this restrictive setting and offers a more realistic description of the tails of the daily return distribution. It may therefore yield more accurate estimates of the quantiles of a portfolio's profit and loss distribution. Our application to forecasting the value at risk of two equity portfolios confirms this expectation.

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In this research, we build on several strands of literature. The first strand is a series of studies in the RV literature extending the univariate heterogeneous autoregressive (HAR) model to the multivariate level. The HAR model, originally suggested by Corsi (2009), is a stationary, restricted AR(22) model that captures the long-range dependence in RV data by means of a cascade of volatility components, which are interpreted as a daily, weekly, and monthly volatility component. Today, it is a standard benchmark for modeling RV with unrivaled forecasting performance; see Corsi et al. (2012) for a review. As an alternative to the HAR model, pure long-memory models belonging to the ARFIMA class have been considered for modeling RV, but their forecasting performance is close to that of HAR-type models; see Baillie (1996), Baillie et al. (1996), Andersen et al. (2003), among others. A nontrivial challenge in constructing a multivariate HAR model for RV, i.e., a RCov model, is to ensure positive-definiteness of predicted covariance matrices. A number of avenues have been pursued concurrently. First, one considers modeling nonlinear transformations of RCov; examples include the Fisher transformation as in Audrino and Corsi (2010), the Cholesky factorization as in Chiriac and Voev (2011), or the matrix log transformation as in Bauer and Vorkink (2011). Second, one models the time series of realized covariances by means of a Wishart process; see Gouriéroux et al. (2009), Bonato et al. (2012), Golosnoy et al. (2012), Jin and Maheu (2012). Third, one may follow the routes of multivariate GARCH and DCC models such as suggested in Colacito et al. (2011), Hansen et al. (2011b), Noureldin et al. (2012), and Bauwens et al. (2012). A fourth strand employs classical factor models; see Hautsch and Kyj (2010) and Bannouh et al. (2012). The RCop model is in the spirit of this research, since the copula parameter, which we infer from RCov and subsequently describe by a time series model, defines – together with the assumptions on the marginals – a multivariate distribution and consequently a well-posed covariance matrix.

The second stream of research our work is related to is the growing literature of dynamic copula models, such as Dias and Embrechts (2004) and Patton (2004, 2006), Chen and Fan (2006), Jondeau and Rockinger (2006), Giacomini et al. (2009), Jin (2010), Christoffersen et al. (2011), Hafner and Manner (2012), Creal et al. (2013), and Härdle et al. (2013). Common to all these approaches is the notion of a copula structure that has time-varying parameters driven by past realizations of the underlying data generating process or by additional exogenous variables such as a latent state factor. By exploiting intra-day data, we uncover a daily time series of implied copula parameters which we model by formulating a time series model. We thus obtain a dynamic conditional copula model for daily returns, where time-variation is governed by the underlying dynamics of RCov measures.

Remarkably, the literature using copulae to model dependency in the context of high-frequency data is scarce. In two early studies, Breymann et al. (2003) and Dias and Embrechts (2004), a copula is directly applied to analyze intraday returns. This is not the purpose of the present investigation. As in De Lira Salvatierra and Patton (2013), our aim is to exploit the intraday information as condensed in the RV measure to improve the modeling of daily returns within a copula framework. In this sense, we follow recent suggestions by Engle and Gallo (2006), Shephard and Sheppard (2010), Hansen et al. (2012), and Hansen et al. (2011b) that combine both low and high-frequency observations in the fashion of GARCH models to describe returns at daily frequency.

The paper is organized as follows. In Section 2, we introduce the RCop model and discuss estimation. The competitor models, which we use for the comparative risk management study, are presented in Section 3. In Section 4, we compare the empirical properties of all models out-of-sample by assessing their one-day ahead value at risk predictions on two portfolios of heavily traded US stocks. Section 5 concludes.

2. The realized copula model

2.1. Estimation of copula parameter implied by realized variance

Copulae have emerged as a convenient way to construct multivariate distributions since they allow a strict separation of the marginal distributions from cross-sectional dependence, which is captured by the copula function; see Joe (1997), Nelsen (2006) and Jaworski et al. (2013) for an introduction to copulae. The main result due to Sklar (1959) states that if F is an arbitrary d-dimensional continuous distribution function of the random variables $X_1, \ldots, X_d$, then the associated copula is unique and defined as a continuous function $C : [0, 1]^d \to [0, 1]$ satisfying the equality

$$C(u_1, \ldots, u_d) = F\{F_1^{-1}(u_1), \ldots, F_d^{-1}(u_d)\}, \quad u_1, \ldots, u_d \in [0, 1], \tag{1}$$

where $F_1^{-1}, \ldots, F_d^{-1}$ are the quantile functions of the corresponding marginal distributions $F_1, \ldots, F_d$. If F belongs to the class of elliptical distributions, this results in a so-called elliptical copula. Most elliptical copulae, however, cannot be given explicitly, because the distribution function F and the inverse marginal distributions F_i usually have integral representations.

A class of copulae overcoming this drawback is the class of Archimedean copulae

$$C_\theta(u_1, \ldots, u_k) = \phi_\theta\{\phi_\theta^{-1}(u_1) + \cdots + \phi_\theta^{-1}(u_d)\}, \quad u_1, \ldots, u_d \in [0, 1], \tag{2}$$

where $\phi_\theta : [0, \infty) \to [0, 1]$, with $\phi_\theta(0) = 1$, $\phi_\theta(\infty) = 0$. The function ϕ_θ is called the generator of the copula, and it usually depends on a single parameter θ. The generator ϕ_θ is required to be d-monotone, i.e., differentiable up to the order $d-2$, with $(-1)^j \phi_\theta^{(j)}(x) \geq 0, j = 0, \ldots, d-2$, for any $x \in [0, \infty)$ and with $(-1)^{d-2}\phi_\theta^{(d-2)}(x)$ being nondecreasing and convex on $[0, \infty)$; see McNeil and Nešlehová (2009). We present some examples of Archimedean copulae and their generators in Table 1; see Nelsen (2006) for more details.

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