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A simple test for a bubble based on growth and acceleration

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ABSTRACT

Time series with bubble-like patterns display an unbalance between growth and acceleration. When growth in the upswing is "too fast", then soon there will be a collapse and the bubble bursts. Such time series thus shows periods where both the first differences and the second differences of the data are positive-valued and in unbalance. For a time series without such bubbles, it can be shown that the variable when properly differenced has a stable mean. A simple test based on recursive residuals can now be used to timely discover whether a series experiences a bubble and also whether a collapse is near. Illustration on simulated data and on two housing prices and the Nikkei index illustrates the practical relevance of the new test. Monte Carlo simulations indicate that the empirical power of the test can be high.

0. Introduction

There is ample interest in economic bubbles, both in academia and in practice. Important work in this area is covered by Diba and Grossman (1988), Evans (1991), Shiller (1981) and West (1987). Usually, economic bubbles are associated with price series (after correction for inflation) showing explosive behavior for a short period, and once the bubble bursts, there is a return to a (much) lower level. Bubbles can occur in almost any price series, and evidence has been documented for stock markets, housing prices, postage stamps prices, art prices, raw materials and many more. Fig. 1 depicts an almost prototypical graph of a time series which displays such a bubble, that is, the Nikkei index, observed annually for 1914–2005, where the bubble burst in 1990.

For analysts it is important to timely diagnose whether a price series is experiencing a bubble-like pattern, and even better, to forecast when the bubble will burst. Recent research has shown that time-series-based methods can be informative to diagnose the current state of affairs. Homm and Breitung (2012) evaluate various econometric testing methods, amongst which is the technique developed in Phillips et al. (2011), and they rank order their empirical performance using Monte Carlo methods. Breitung and Kruse (2013) build on these results to propose monitoring procedures. In this paper I add a new and simple test for a bubble to the analyst's toolkit. The test aims at predicting when the bubble bursts, given that there is a bubble-like pattern in the data. It can be used recursively.

The new test is based on the notion of the balance between acceleration and growth of a time series, where growth is associated with first differences and acceleration is associated with the first differences of these first differences. Looking again at Fig. 1, it is clear that in the years before 1990, the Nikkei data not only showed a positive growth, but there also was a long period in which this growth amplified. This phenomenon of positive growth and positive acceleration can be coined as positive feedback. Such a feedback drives the data to ever higher levels, mimicking explosive behavior. In contrast, when growth and acceleration are at balance, it can be shown that a specifically transformed time series should have a stable mean

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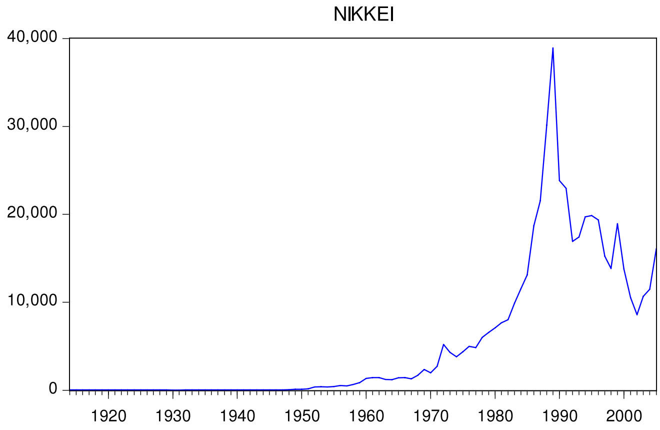

Fig. 1. The annual Nikkei index 1914–2005.

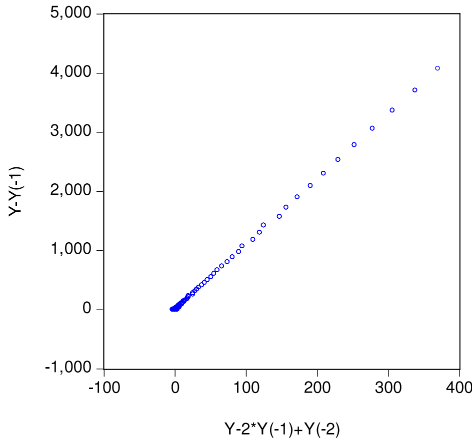

Fig. 2a. Simulated data for an AR(1) process with $\rho = 1.1$.

and would not show bubble-like patterns. This will be outlined in Section 2. This stability implies that a simple test based on one-step-ahead forecast errors can be used to recursively test for deviations from stability. The test can even be used to give warning signals for an upcoming collapse. The usefulness of this test will further be illustrated on simulated data and on three actual time series in Section 3. Monte Carlo simulations show that the test has the proper size and also that it can have substantial power. Section 4 concludes with suggestions for further research.

1. The main idea

Consider an inflation-corrected price series y_t observed for $t = 1, 2, \ldots, T$, and assume for the moment that the series has at most one zero-frequency unit root, that is, at most the series is I(1). Bubble-like behavior, like in Fig. 1, entails that for some period the growth of y_t is positive and also that the growth of the growth (to be called: acceleration) is positive, after which there is a collapse. In time series notation, while using the familiar lag operator L, this means that $(1 - L)y_t$ and $(1 - L)^2 y_t$ both have positive values at the same moment in time. Such positive feedback makes the time series data explode. The end of such an explosive period can be marked by the burst of the bubble, where after the collapse the data move towards a lower price level (at least for a while). Fig. 2a illustrates this positive feedback for an autoregression of order 1 [AR(1)] with parameter ρ equal to 1.10, where a scatter plot of $(1-L)y_t$ versus $(1-L)^2 y_t$ is presented. Clearly, during the noticeable explosive stage when time proceeds, the data start to quickly move away from the initial cloud of data points. Fig. 2b shows that this phenomenon does not occur for stationary data, as the points in the scatter plot are tied towards the cloud of points for an AR(1) with a ρ equal to 0.8.

Hence, for stationary series the two derivative series are at balance, that is, there seems a stable relationship between $(1-L)y_t$ and $(1-L)^2 y_t$. Such a stable relationship implies that another derivative series does not display bubbles, as will be shown now, which effectively means that the series has a stable mean. Consider an ARMA time series process for y_t, where this series can have a single zero frequency unit root, that is, $\varphi(L)\, y_t = \theta(L)\varepsilon_t$ where ε_t is a white noise process with mean zero and variance σ_ε^2. The regression line connecting the data of $(1 - L)y_t$ with $(1 - L)^2 y_t$ (like in Figs. 2a and 2b) can be

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