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On conditional covariance modelling: An approach using state space models

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ABSTRACT

A novel approach to conditional covariance modelling is introduced in the context of multivariate financial time series analysis. In particular, a class of multivariate generalized autoregressive conditional heteroscedasticity models is proposed. The suggested modelling technique is based on a specific dynamic orthogonal transformation derived by the LDL factorization of the conditional covariance matrix. An observed time series is transformed into a particular form that can be further treated by means of a discrete-time state space model under corresponding assumptions. The calibration can be performed by the associated Kalman recursive formulas, which are numerically effective. The introduced procedure has been investigated by extensive Monte Carlo experiments and empirical financial applications; it has been compared with other methods commonly used in this framework. The outlined methodology has demonstrated its capabilities, and it seems to be at least competitive in this field of research.

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1. Introduction

Analysis of conditional covariances and correlations is undoubtedly an important part of multivariate financial time series modelling. From a general theoretical perspective, examining a time-varying behaviour of conditional covariances with eventual regard to modelling conditional correlations is indeed worth of interest. One can primarily mention at least two reasons amongst others. Firstly, financial time series generally require applying specific model concepts that respect their special character. For instance, various (multivariate) GARCH processes can be highlighted here. Secondly, several constraints are naturally connected with the considered quantities of interest. In greater detail, the conditional covariance (correlation) matrix must be symmetric and positive definite. Additionally, the conditional correlation matrix must have unit diagonal elements. Indisputably, such requirements might bring tough limitations into calibration, especially in the case of higher dimensions. Therefore, such restrictions must be seriously taken into account, e.g. by implementing various representations which simplify or completely eliminate them, see below.

Modelling conditional correlations is inherently linked to modelling conditional covariances. In general, one can essentially distinguish between two approaches: (i) the *direct* one, where a model representation of conditional covariances involves an explicit expression of conditional correlations, and (ii) the *indirect* one, where conditional correlations are indirectly calculated as normalizations of conditional covariances.

Moreover, the introduced topic is also worth of interest from the practical point of view. Particularly, conditional correlations are crucial inputs for many tasks of technical analysis or financial, portfolio, and risk management, e.g. an asset

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allocation, a construction of an optimal portfolio, or a hedging problem. The relevance of the discussed issue is evident from the substantial body of literature in this research field. Many academically or practically oriented publications treat this topic from many viewpoints. In particular, see the works by Bollerslev (1990), Engle (2002), Bauwens et al. (2006), Engle and Colacito (2006), Engle (2009), Alp and Demetrescu (2010), Rossi and Spazzini (2010), Aielli (2011), or Hafner and Reznikova (2012), and the references given therein.

In this article, a class of multivariate generalized autoregressive conditional heteroscedasticity models is proposed. The suggested technique is rested on a specific time-varying orthogonal transformation delivered by the LDL decomposition of the conditional covariance matrix. This transformation decomposes multivariate time series; therefore, it enables to apply the instruments of the linear discrete-time state space methods under some modelling circumstances. In addition, the corresponding Kalman recursions provide a numerically effective way of calibration. Furthermore, the introduced approach is primarily studied in the context of conditional correlations (compare with Engle, 2002). Its performance is analysed by means of a simulation study and empirical applications.

The paper is organized as follows. Section 2 introduces a general multivariate model framework. It can be regarded as a straightforward analogy of univariate conditional heteroscedastic models. Section 3 presents the proposed modelling technique in detail. Section 4 briefly surveys other estimators of conditional covariances. Section 5 describes a Monte Carlo study, which compares the accuracy of the introduced technique with others. Section 6 investigates two different empirical examples: (i) the first one studies bivariate correlations between stocks and bonds, (ii) the second one analyses correlation links in a portfolio of six European currencies. Finally, Section 7 contains conclusions.

2. Model framework

Consider a multivariate stochastic vector process $\{\mathbf{X}_t\}_{t\in\mathbb{Z}}$ of dimension $(n \times 1)$. Denote $\mathcal{F}_t$ the σ-algebra generated by observed time series up to and including time t, i.e. $\mathcal{F}_t = \sigma(\mathbf{X}_s, s \le t)$ is the smallest σ-algebra with respect to which $\mathbf{X}_s$ is measurable for all $s \le t, s, t \in \mathbb{Z}$.

In this framework, assume the following model

$$\mathbf{X}_t = \mathbf{H}_t^{1/2}\mathbf{Z}_t, \tag{1}$$

where $\mathbf{H}_t = (h_{ij,t})_{i,j=1}^n$ is the $(n \times n)$ positive definite conditional covariance matrix of $\mathbf{X}_t$ given $\mathcal{F}_{t-1}$. Furthermore, one supposes that $\{\mathbf{Z}_t\}$ is an $(n \times 1)$ i.i.d. stochastic vector process with the moments $\mathsf{E}(\mathbf{Z}_t) = \mathbf{0}$ and $\mathsf{cov}(\mathbf{Z}_t) = \mathbf{I}_n$, where $\mathbf{I}_n$ denotes the identity matrix of order n.

In the model (1), the conditional and unconditional moments of $\mathbf{X}_t$ can be easily calculated:

$$\mathsf{E}(\mathbf{X}_t|\mathcal{F}_{t-1}) = \mathbf{0}, \qquad \mathsf{cov}(\mathbf{X}_t|\mathcal{F}_{t-1}) = \mathbf{H}_t^{1/2}(\mathbf{H}_t^{1/2})^\top = \mathbf{H}_t, \tag{2}$$

$$\mathsf{E}(\mathbf{X}_t) = \mathbf{0}, \qquad \mathsf{cov}(\mathbf{X}_t) = \mathsf{E}(\mathbf{H}_t), \qquad \mathsf{cov}(\mathbf{X}_t, \mathbf{X}_{t+h}) = \mathbf{0}, \quad h \neq 0. \tag{3}$$

Apparently, $\mathbf{H}_t^{1/2}$ is any $(n\times n)$ positive definite matrix such that $\mathbf{H}_t$ is the conditional covariance matrix of $\mathbf{X}_t$ given $\mathcal{F}_{t-1}$. The considered decomposition of $\mathbf{H}_t$ may be delivered e.g. by the Cholesky factorization as it is common in the literature, see e.g. Engle (2002). Moreover, $\mathbf{R}_t$ (the conditional correlation matrix of $\mathbf{X}_t$ given $\mathcal{F}_{t-1}$) can be obtained by a simple normalization of the conditional covariance matrix.

3. Conditional covariances via state space modelling

Following the algebraic theory, each real symmetric positive definite matrix has a unique LDL decomposition (Harville, 1997). Let the conditional covariance matrix $\mathbf{H}_t$ have the LDL reparameterization in its standard form, i.e.

$$\mathbf{H}_t = \mathbf{L}_t\mathbf{D}_t\mathbf{L}_t^\top [= (\mathbf{L}_t\mathbf{D}_t^{1/2})(\mathbf{L}_t\mathbf{D}_t^{1/2})^\top = \mathbf{H}_t^{1/2}(\mathbf{H}_t^{1/2})^\top], \tag{4}$$

where $\mathbf{L}_t = (\ell_{ij,t})_{i,j=1}^n$ is an $(n \times n)$ lower triangular matrix with the unit diagonal and $\mathbf{D}_t$ is an $(n \times n)$ diagonal matrix with positive elements $d_{ii,t}$ on its diagonal. In particular, $\det(\mathbf{L}_t) = 1$, $\mathbf{L}_t$ is invertible, and the inverted matrix $\mathbf{L}_t^{-1} = (\ell_t^{ij})_{i,j=1}^n$ is also an $(n \times n)$ lower triangular matrix with unit diagonal elements. It is noteworthy that the decomposition (4) requires no parameter constraints for $\mathbf{H}_t$ being symmetric and positive definite since this is guaranteed by the LDL structure.

The LDL factorization (4) delivers uniquely determined recurrent relations for the elements of $\mathbf{L}_t$ and $\mathbf{D}_t$ (Harville, 1997). One can easily derive the following formulas for conditional covariances and correlations:

$$h_{ii,t} = \mathsf{var}(X_{i,t}|\mathcal{F}_{t-1}) = \sum_{v=1}^{i} \ell_{iv,t}^2 d_{vv,t}, \quad i = 1, \ldots, n, \tag{5}$$

$$h_{ij,t} = \mathsf{cov}(X_{i,t}, X_{j,t}|\mathcal{F}_{t-1}) = \sum_{v=1}^{j} \ell_{iv,t}\ell_{jv,t}d_{vv,t}, \quad \text{for } j < i, \ i = 2, \ldots, n, \tag{6}$$

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