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Matrix exponential stochastic volatility with cross leverage

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ABSTRACT

A multivariate stochastic volatility model with the dynamic correlation and the cross leverage effect is described and its estimation using Markov chain Monte Carlo is proposed. The time-varying covariance matrices are guaranteed to be positive definite by using a matrix exponential transformation. Of particular interest is our approach for sampling a set of latent matrix logarithm variables from their conditional posterior distribution, where we construct the proposal density based on an approximating linear Gaussian state space model. The proposed model and its extensions with fat-tailed error distribution are applied to trivariate returns data (daily stocks, bonds, and exchange rates) of Japan. Further, a model comparison is conducted including constant correlation multivariate stochastic volatility models with leverage and diagonal multivariate GARCH models.

1. Introduction

Over the last several decades, there has been a great deal of interest in modeling volatilities of multivariate stock market returns. The examples are multivariate generalized autoregressive conditional heteroskedasticity (GARCH) models (see the review of Bauwens et al., 2006), multivariate stochastic volatility (SV) models (see the review of Asai et al., 2006, Chib et al., 2009) and realized covariance models (see e.g. Golosnoy et al., 2012). The realized covariance model uses the high-frequency data to estimate covariance matrices and regard them as observed covariance matrices, while they are latent variables in GARCH and SV models.

Various multivariate volatility models have been proposed in the literature to describe the dynamic properties of the covariance matrices such as the volatility clustering, the dynamic correlations, and the leverage effects. The DCC models (Engle, 2002) and BEKK models (Engle and Kroner, 1995) are examples in multivariate GARCH models. In the multivariate SV models, it is difficult to keep the covariance matrices positive definite. To overcome this difficulty, several reparameterization methods are considered in Yu and Meyer (2006), Tsay (2005), and Jungbacker and Koopman (2006). The Cholesky decomposition of the covariance matrix is also considered in Lopes et al. (2012) and Loddo et al. (2011). Alternative approaches using Wishart process are also proposed for both GARCH and SV models with or without realized covariance matrices (Philipov and Glickman, 2006; Gourieroux et al., 2009; Golosnoy et al., 2012; Jin and Maheu, 2013).

However, there have been still few previous works on the multivariate volatility models with both dynamic correlations and cross leverage effects. Cross leverage refers to the correlation between the ith asset return at time t and the function of jth asset volatility at time $t+1$ (when $i=j$, we simply call it a leverage effect). Thus, to model these properties of covariance matrices, this paper considers the matrix logarithm transformation which is known useful to model positive

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definite matrices in a flexible way. Since the seminal work of Chiu et al. (1996), the matrix exponential model for the covariance matrix has been applied to the spatial model to simplify the calculation of log-likelihood functions (LeSage and Pace, 2007), and is extended to the GARCH model (Kawakatsu, 2006), the SV model (Asai et al., 2006) and the realized covariation model (Bauer and Vorkink, 2010; Sheppard, 2007) for multivariate financial time series.

We consider the general multivariate volatility model using the matrix exponential SV model with cross leverage effects and propose a computational algorithm. This is a generalization of Ishihara and Omori (2012) who propose the following multivariate stochastic volatility (MSV) model with cross-asset leverage effect of the form

$$\boldsymbol{y}_t = \mathrm{diag}\left(\exp(\alpha_{1t}/2), \ldots, \exp(\alpha_{pt}/2)\right) \boldsymbol{\varepsilon}_t, \tag{1}$$

$$\boldsymbol{\alpha}_{t+1} = \boldsymbol{\Phi}\boldsymbol{\alpha}_t + \boldsymbol{\eta}_t, \tag{2}$$

$$\left(\boldsymbol{\varepsilon}_t', \boldsymbol{\eta}_t'\right)' \sim \mathcal{N}_{2p}(\mathbf{0}, \boldsymbol{\Sigma}), \tag{3}$$

where $\boldsymbol{y}_t = (y_{1t}, \ldots, y_{pt})'$, $\boldsymbol{\alpha}_t = (\alpha_{1t}, \ldots, \alpha_{pt})'$, $\boldsymbol{\Phi} = \mathrm{diag}(\phi_1, \ldots, \phi_p)$ and $\mathcal{N}_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ denotes the p-dimensional normal distribution with mean $\boldsymbol{\mu}$ and variance $\boldsymbol{\Sigma}$. This is fairly general in the sense that there is no restriction imposed on the covariance matrix $\boldsymbol{\Sigma}$, while, in the previous literature, various parameter restrictions are imposed (e.g. Asai and McAleer, 2006, Asai and McAleer, 2009, Chan et al., 2006, and Daníelsson, 1998) to estimate parameters based on the Monte Carlo likelihood. We, further, model the dynamic covariance matrices (dynamic variances and correlations) using a matrix logarithm transformation. Since it is difficult to implement a maximum likelihood estimation for our proposed model without imposing restrictions on parameters, we take the Bayesian approach and estimate posterior distributions of model parameters using the Markov chain Monte Carlo (MCMC) method. It is well-known that MCMC algorithms sometimes suffer from the sampling inefficiency problem in stochastic volatility models. As discussed in Ishihara and Omori (2012), the simple sampling algorithm for the latent covariance matrices is found to be inefficient in the sense that the generated MCMC samples of latent volatility variables are highly autocorrelated. They showed that the single-move sampler which samples one volatility variable given others is highly inefficient and proposed the efficient multi-move sampler (block sampler) which divides the vector of all latent variables into blocks and samples one block given other blocks based on Omori and Watanabe (2008). Thus we construct the multi-move sampler for our matrix exponential model and compare with the alternative simple sampling algorithm.

The rest of the paper is organized as follows. In Section 2, we introduce a matrix exponential stochastic volatility model with cross leverage effects. The Bayesian estimation method and the associated particle filter for calculating likelihood functions are described in Section 3. And in Section 4, the empirical studies are given using the trivariate asset returns data (stock indices, bond indices and foreign exchange rates). We conduct a model selection among the proposed model, extended models with fat-tailed error distribution, some constant correlation multivariate SV models, and some diagonal BEKK models. Section 5 concludes the paper. The efficiency of our proposed algorithm using the simulated data is investigated in the working paper version.

2. Matrix exponential stochastic volatility

This section proposes the matrix exponential stochastic volatility (MESV) model with cross leverage effects. The MESV model is based on the matrix exponential transformation as below. A matrix exponential is widely studied in the context of multidimensional differential equations and Lie algebra. The statistical applications of the matrix exponential transformation are given, for example, in Chiu et al. (1996), and Kawakatsu (2006). For any $p \times p$ matrix $\mathbf{A}$, the matrix exponential is defined by the following power series expansion

$$\exp(\mathbf{A}) \equiv \sum_{s=0}^{\infty} \frac{1}{s!}\mathbf{A}^s,$$

where the series converges absolutely if all eigenvalues of $\mathbf{A}$ are finite. (See e.g. Abadir and Magnus, 2005 for various properties of the matrix exponential transformation.) For any real symmetric positive definite matrix $\mathbf{C}$, there exists a real symmetric $p \times p$ matrix $\mathbf{A}$ such that $\mathbf{C} = \exp(\mathbf{A})$, and the matrix $\mathbf{A}$ is obtained by the matrix logarithm transformation. Conversely, for any real symmetric matrix $\mathbf{A}$, $\mathbf{C} = \exp(\mathbf{A})$ is a symmetric positive definite matrix (Chiu et al., 1996). If $\mathbf{A}$ is a $p \times p$ real symmetric matrix, there exists a $p \times p$ orthogonal matrix $\mathbf{U}$ and a diagonal matrix $\boldsymbol{\Lambda}$ such that $\mathbf{A} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}'$ and

$$\exp(\mathbf{A}) = \mathbf{U}\left(\sum_{s=0}^{\infty} \frac{1}{s!}\boldsymbol{\Lambda}^s\right)\mathbf{U}' = \mathbf{U}\exp(\boldsymbol{\Lambda})\mathbf{U}'.$$

Now let $\boldsymbol{y}_t = (y_{1t}, \ldots, y_{pt})'$ denote the p-dimensional asset return vector at time t, and let $\mathbf{H}_t$ denote the matrix logarithm of the variance–covariance matrix of $\boldsymbol{y}_t$. The MESV model with leverage effects is given by

$$\boldsymbol{y}_t = \exp(\mathbf{H}_t/2)\boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t \sim \text{i.i.d. } \mathcal{N}_p(\mathbf{0}, \mathbf{I}_p),\ t = 1, \ldots, n. \tag{4}$$

$$\mathbf{H}_{t+1} = \mathbf{M} + \tilde{\boldsymbol{\Phi}} \odot (\mathbf{H}_t - \mathbf{M}) + \mathbf{E}_t, \tag{5}$$

$$\begin{pmatrix}\boldsymbol{\varepsilon}_t \\ \boldsymbol{\eta}_t\end{pmatrix} \sim \text{i.i.d. } \mathcal{N}_{p+q}(\mathbf{0}, \boldsymbol{\Sigma}), \quad \boldsymbol{\Sigma} = \begin{pmatrix}\mathbf{I}_p & \boldsymbol{\Sigma}_{\varepsilon\eta} \\ \boldsymbol{\Sigma}_{\eta\varepsilon} & \boldsymbol{\Sigma}_{\eta\eta}\end{pmatrix},\ t = 1, \ldots, n-1, \tag{6}$$

$$\boldsymbol{h}_1 \sim \mathcal{N}_q\left(\boldsymbol{\mu}, \boldsymbol{\Sigma}_0\right), \tag{7}$$