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A Bayesian non-parametric approach to asymmetric dynamic conditional correlation model with application to portfolio selection

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ABSTRACT

A Bayesian non-parametric approach for modeling the distribution of multiple returns is proposed. More specifically, an asymmetric dynamic conditional correlation (ADCC) model is considered to estimate the time-varying correlations of financial returns where the individual volatilities are driven by GJR-GARCH models. This composite model takes into consideration the asymmetries in individual assets' volatilities, as well as in the correlations. The errors are modeled using a Dirichlet location–scale mixture of multivariate Normals allowing for a flexible return distribution in terms of skewness and kurtosis. This gives rise to a Bayesian non-parametric ADCC (BNP-ADCC) model, as opposed to a symmetric specification, called BNP-DCC. Then these two models are compared using a sample of Apple Inc. and NASDAQ Industrial index daily returns. The obtained results reveal that for this particular data set the BNP-ADCC outperforms the BNP-DCC model. Finally, an illustrative asset allocation exercise is presented.

1. Introduction

Modeling the stylized features of the assets' returns has been extensively researched for decades and the topic yet remains of great interest. The ARCH-family models, first introduced by Engle (1982) and then generalized by Bollerslev (1986), are without doubt the most analyzed and used in practice to explain time-varying volatilities, see Bollerslev et al. (1992), Bollerslev et al. (1994), Engle (2002b), Teräsvirta (2009) and Tsay (2010).

Empirical evidence shows that returns and volatilities exhibit three types of asymmetries. The first two asymmetries are present in the dynamics of volatilities and correlations: these respond to changes in returns in an asymmetric manner that depends on the sign of the return. The third type of asymmetry is present in the unconditional distribution of the returns and is modeled via distributional assumptions for the error term.

Asymmetry in the volatility response to the changes in the returns, sometimes also called "leverage effect", was first introduced by Black (1976). It means that negative shocks to the returns have a stronger effect on volatility than positive ones of the same magnitude. When dealing with multiple returns, one must also take into consideration the mutual dependence between them, see Bauwens et al. (2006), Silvennoinen and Teräsvirta (2009) and Tsay (2010). In particular, conditional correlation models, first proposed by Engle (2002a), Tse and Tsui (2002) and Christodoulakis and Satchell (2002), play an

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important role because there is evidence that conditional correlations between returns are time dependent. More recently, Cappiello et al. (2006) have proposed Asymmetric Dynamic Conditional Correlation (ADCC) model for time-varying correlations. Cappiello et al. (2006) argue that correlations between asset returns might be higher after a negative return than after a positive one of the same size. These two types of asymmetries govern the deterministic evolution of volatilities and correlations.

On the other hand, the third type of asymmetry – the unconditional one – can be modeled via the distribution of the returns. Many of the GARCH models, univariate or multivariate, rely on Gaussianity assumption for the error term. However, the traditional premises of Normal distribution may be rather restrictive because the empirical unconditional distribution of returns, as mentioned before, is usually slightly skewed (asymmetric) and fat-tailed, see Rossi and Spazzini (2010), for example. Alternative parametric choices, such as the Student-t density, see Fiorentini et al. (2003), the skew-Student-t distribution, see Bauwens and Laurent (2005), or finite mixtures of Gaussian distributions, see e.g. Ausín and Galeano (2007) and Galeano and Ausín (2010), have been proposed in the literature and they usually improve the fit of GARCH models. However, all of them require the assumption of a certain parametric model. An alternative is to abandon the parametric setting altogether and consider a Dirichlet Process Mixture (DPM) model of Gaussian distributions, first introduced by Lo (1984). This is a very flexible model which can be viewed as a location–scale mixture of Gaussian distributions and is capable of modeling the Gaussian, Student-t, logistic, double exponential, Cauchy and generalized hyperbolic distributions, among others, see e.g. Tokdar (2006) and Mencía and Sentana (2009).

Therefore, in this paper we consider an ADCC model for time-varying correlations with GJR-GARCH for individual volatilities (Glosten et al., 1993) and a DPM model for the return innovations, resulting into Bayesian non-parametric ADCC model (BNP-ADCC). We follow closely the works of Kalli et al. (2013) and Ausín et al. (2014), who have applied the DPM models for univariate GJR-GARCH and Jensen and Maheu (2013), who have used DPM models for the multivariate symmetric DVEC (Ding and Engle, 2001). Non-parametric time-varying volatility models have been of great interest in the recent literature, in both, GARCH and Stochastic Volatility setting, see Jensen and Maheu (2010, 2013, 2014), Delatola and Griffin (2011, 2013), Kalli et al. (2013) and Ausín et al. (2014). For a survey on Bayesian inference methods for univariate and multivariate GARCH models see Virbickaitė et al. (in press).

Jensen and Maheu (2013) have established the superiority of non-parametric errors in MGARCH models as compared to most commonly used parametric distributions, such as Gaussian and Student-t. In this paper, we carry out an extensive comparison between our proposed BNP-ADCC model and its fully symmetric version, BNP-DCC, both with non-parametric errors.

The improved fit of the model to multiple financial time series can be applied to risk management problems, such as, portfolio optimization, for example. In this paper, we propose a Bayesian method which provides the posterior distributions of the one-step-ahead optimal portfolio weights, which are more informative than simple point estimates. The Bayesian approach also helps to deal with parameter uncertainty in portfolio decision problems, see e.g. Jorion (1986), Greyserman et al. (2006), Avramov and Zhou (2010) and Kang (2011), among others. This is in contrast with the usual maximum likelihood estimation approach, which assumes a "certainty equivalence" viewpoint, where the sample estimates are treated as the true values, which is not in general correct and has been criticized in a number of papers, see Brown (1978) and Jorion (1986), among others.

Therefore, the main contribution of this work is the proposal of a Bayesian non-parametric method for explaining the asymmetric dynamics of the assets' returns via a BNP-ADCC model. We carry out a comparison with a fully symmetric BNP-DCC model to examine if by considering the asymmetric volatility and correlation response we can improve the prediction accuracy. Also, we present an application of Bayesian non-parametric techniques in portfolio decision problems and explore the differences in uncertainty between the two models. This paper extends the work by Ausín et al. (2014) to the multivariate framework and the recent work by Jensen and Maheu (2013) to the asymmetric setting. Also, differently from the work of Jensen and Maheu (2013), we always assume a conjugate prior specification and we use a different sampling approach.

The outline of the paper is as follows: Section 2 describes the model, inference and prediction from a Bayesian perspective. Section 3 introduces the time-varying portfolio optimization problem. Section 4 presents a short simulation study. Section 5 illustrates the proposed approach using a real data example, compares the models and solves a portfolio allocation problem. Section 6 concludes.

2. Model, inference and prediction

This section describes the asymmetric dynamic conditional correlation GJR-GARCH model used for volatilities and correlations and the DPM specification for the error term, resulting in the BNP-ADCC model. Then we provide a detailed explanation of the implementation of Bayesian non-parametric inference and the methodology of obtaining predictive densities of returns and volatilities.

2.1. The Bayesian non-parametric ADCC model

Financial returns usually exhibit asymmetries in individual volatilities and in conditional correlations. Therefore, on the one hand, we choose the GJR-GARCH model proposed by Glosten et al. (1993) for individual returns to incorporate

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