Computational Statistics and Data Analysis 94 (2016) 86–97

Contents lists available at ScienceDirect

Computational Statistics and Data Analysis

journal homepage: www.elsevier.com/locate/csda

Diagnostic checking of the vector multiplicative error model

F.C. Ng *, W.K. Li, Philip L.H. Yu

Department of Statistics and Actuarial Science, The University of Hong Kong, Pokfulam Road, Hong Kong, China

ARTICLE INFO

Article history:
Received 17 October 2014
Received in revised form 6 June 2015
Accepted 25 July 2015
Available online 13 August 2015

Keywords:
Multivariate portmanteau test
Autoregressive conditional duration
GARCH model
Realized volatility

ABSTRACT

In many situations, we may encounter time series that are non-negative. Examples include trading duration, volume transaction and price volatility in finance, waiting time in a queue in social sciences, and daily/hourly rainfall in natural sciences. The vector multiplicative error model (VMEM) is a natural choice for modeling such time series in a multivariate framework. Despite the popularity and extensive use of the model, very little work has been done on the area of diagnostic checking which however provides useful information about the adequacy of model fitting. In this paper, the asymptotic distribution of residual autocorrelations is derived and used to devise a new multivariate portmanteau test for diagnostic checking. Simulation studies are performed to assess the performance of the asymptotic result in finite samples. An empirical example is also given to demonstrate that the commonly used goodness-of-fit test may lead to a misleading result in the case of the VMEM.

1. Introduction

In many situations, we may encounter time series that are non-negative. For example, in financial time series analysis, the duration between consecutive trades, volume transaction, realized volatility and daily price range of an asset are all non-negative. The multiplicative error model (MEM) (introduced by Engle, 2002) and its multivariate extension, the vector multiplicative error model (VMEM), are standard approaches to model such non-negative-valued time series, especially when these time series share similar persistence and clustering features as the squared returns. A popular special case of this model class is the autoregressive conditional duration (ACD) model for financial durations (reviewed in Pacurar, 2008) that was first proposed by Engle and Russell (1998). Since then, multiplicative error specifications have been adopted for modeling different kinds of financial data such as realized volatility (Engle and Gallo, 2006; Lanne, 2006; Cipollini et al., 2012), squared return (Engle and Gallo, 2006; Hautsch, 2008), trading volume (Manganelli, 2005), price range (Chou, 2005; Engle and Gallo, 2006), and implied volatility (Ahoniemi and Lanne, 2009). Ding (2012) gave an extensive literature review on the MEM and VMEM. As pointed out in Koul et al. (2012), potential applications of the MEM family in economics, social sciences and natural sciences include modeling demand for electricity, waiting time in a queue and daily/hourly rainfall.

The VMEM, a model for multivariate time series with non-negative components, has been proposed due to the demand for joint modeling of different volatility measures. For example, Engle and Gallo (2006) proposed a three-dimensional VMEM to jointly model realized volatility, absolute return and price range. In order to simplify the estimation procedure, the authors assumed that different components of the innovation are independent to each other and the conditional expectations of one process are related to the others by means of the lagged realized values but not the lagged conditional expectations. However, these assumptions seem to be restrictive and unrealistic. Cipollini et al. (2007) and Ahoniemi and Lanne (2009) suggested the use of multivariate Gamma distribution to allow dependence between different innovation's components,

* Corresponding author.
E-mail address: patng324@connect.hku.hk (F.C. Ng).

http://dx.doi.org/10.1016/j.csda.2015.07.012

but it complicates the estimation considerably. On the other hand, Cipollini et al. (2007) also proposed an alternative model that uses copula to construct the joint conditional distribution of the innovation. However, the use of copula is often under question in finance and the choice of a copula function seems to be arbitrary. Very often, the major issue of the analysis is the dynamics of the conditional expectations but not the distributional specification of the innovation. Motivating from this idea, Cipollini et al. (2012) proposed a semiparametric specification of the VMEM which bypasses the full specification of innovation's conditional distribution. The model is flexible enough to describe the interaction between different processes and at the same time keeps the estimation tractable.

Another application of the VMEM is joint modeling of different components of realized volatility. Realized volatility is often decomposed into continuous and jump components in order to deal with outliers which are assumed to carry some valuable information (see e.g. Barndorff-Nielsen and Shephard, 2004; Bollerslev et al., 2009; Andersen et al., 2011; Ding, 2012). Ding (2012) defined the jump component as the division of realized volatility by quadpower volatility which acts as the continuous component. Since both the continuous and jump components are non-negative, a bivariate VMEM is used to jointly model these two components.

Diagnostic checking tools are available for MEMs, mainly in the context of ACD models. Li (1991) considered diagnostic checking for time series with conditional generalized linear model distribution. Following the work of Li and Mak (1994), Li and Yu (2003) extended the model checking step of the Box–Jenkins methodology in ARMA processes (Box and Pierce, 1970) to ACD models by deriving the asymptotic distribution of residual autocorrelations when the innovation is exponentially distributed, which results in a portmanteau test statistic. Meitz and Teräsvirta (2006) devised a class of Lagrange multiplier (LM) tests against various forms of misspecification of the conditional expectations and showed that the LM test of no ACD effect on residuals is asymptotically equivalent to that proposed by Li and Yu (2003). Duchesne and Pacurar (2008) made use of a kernel spectral density estimator of the residuals to construct some adequacy tests for ACD models. Chen and Hsieh (2010) proposed a set of generalized moment tests for the conditional expectations based on the quasi-maximum exponential likelihood method. Hong and Lee (2011) developed a class of generalized spectral derivative tests based on the generalized spectrum concept. Koul et al. (2012) considered the diagnostic checking problem of MEMs with Markov structure. All the above work deal with univariate MEMs.

For the VMEM, the only diagnostic test employed in most studies (e.g. Manganelli, 2005; Engle and Gallo, 2006; Hautsch, 2008; Cipollini et al., 2012) was the Box–Pierce–Ljung type portmanteau test (Box and Pierce, 1970; Ljung and Box, 1978; Hosking, 1980) applied to the residuals or squared residuals. From the result of this paper, this approach is questionable since the Box–Pierce statistic may not follow the usual χ^2 distribution asymptotically under the null hypothesis. A similar criticism has been pointed out in the univariate case, see Li and Yu (2003) and the discussion in Duchesne and Pacurar (2008). In fact, in a related context, it has been shown that the Box–Pierce–Ljung type test is invalid for GARCH models (Li and Mak, 1994) and multivariate GARCH models (Ling and Li, 1997).

For diagnostic checking of multivariate ARMA models, Li and McLeod (1981) constructed a multivariate portmanteau test by defining the residual autocorrelation matrices and deriving their asymptotic distribution. Ling and Li (1997) adopted a similar method to the case of multivariate GARCH models. In this paper, following the approach of Li and McLeod (1981) and Ling and Li (1997), we derive the asymptotic distribution of residual autocorrelation matrices in the VMEM and devise a new multivariate portmanteau test for diagnostic checking. Similar to Li and Yu (2003), the result of this paper provides an extension of the Box–Jerkins methodology to the more general VMEM. Interestingly, under a semiparametric setting, our result encompasses a special case in which the proposed test statistic is equivalent to that proposed by Li and Yu (2003) which is based on a parametric specification. Since we do not assume any distributional assumption of the innovation except some very weak moment conditions, the result of this paper is robust with regard to the conditional distribution of the innovation.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 derives the asymptotic distribution of residual autocorrelation matrices in the VMEM and gives the new multivariate portmanteau test statistic that should be useful for diagnostic checking. The relationship between the proposed test statistic and the Li and Yu (2003) test statistic is discussed. Some simulation experiments are performed in Section 4 to assess the performance of the asymptotic result in finite samples and an empirical example is also given in Section 5 to demonstrate that the commonly used multivariate portmanteau test may lead to a misleading result in the case of the VMEM. Finally, we conclude in Section 6.

2. The model

Let $\boldsymbol{x}_t = (x_{1t}, \dots, x_{kt})^{\mathrm{T}}$ be a k-dimensional stationary and ergodic vector time series with non-negative components. The VMEM with a semiparametric specification (Cipollini et al., 2012) for $\boldsymbol{x}_t$ is defined as

$$\boldsymbol{x}_t = \boldsymbol{\mu}_t \odot \boldsymbol{\varepsilon}_t = \mathrm{diag}(\boldsymbol{\mu}_t)\boldsymbol{\varepsilon}_t, \tag{1}$$

where $\odot$ is the Hadamard product and $\mathrm{diag}(\boldsymbol{a})$ denotes a diagonal matrix with main diagonal equals vector $\boldsymbol{a}$. The innovation series $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \dots, \varepsilon_{kt})^{\mathrm{T}}$ is a sequence of independent and identically distributed random vectors defined over a $[0, +\infty)^k$ support, with mean vector $\mathbf{1}_k$ and a general covariance matrix $\boldsymbol{\Sigma} = (\sigma_{ij})_{k\times k}$ (a nuisance parameter); $\mathbf{1}_k$ is a $k \times 1$ vector with all entries equal 1. Vector $\boldsymbol{\mu}_t = (\mu_{1t}, \dots, \mu_{kt})^{\mathrm{T}} = E(\boldsymbol{x}_t \mid \mathcal{F}_{t-1})$ represents the conditional expectation of $\boldsymbol{x}_t$ given $\mathcal{F}_{t-1}$. It depends only on $\mathcal{F}_{t-1}$ and $\boldsymbol{\theta} = (\theta_1, \dots, \theta_p)^{\mathrm{T}}$, where $\mathcal{F}_{t-1}$ is a σ-field generated by $\{\boldsymbol{x}_{t-1}, \boldsymbol{x}_{t-2}, \dots\}$ and $\boldsymbol{\theta}$ is a $p \times 1$ parameter vector ruling the dynamics of $\boldsymbol{\mu}_t$; and is assumed to have continuous second-order derivatives almost surely.