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Interaction models for functional regression

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ABSTRACT

A functional regression model with a scalar response and multiple functional predictors is proposed that accommodates two-way interactions in addition to their main effects. The proposed estimation procedure models the main effects using penalized regression splines, and the interaction effect by a tensor product basis. Extensions to generalized linear models and data observed on sparse grids or with measurement error are presented. A hypothesis testing procedure for the functional interaction effect is described. The proposed method can be easily implemented through existing software. Numerical studies show that fitting an additive model in the presence of interaction leads to both poor estimation performance and lost prediction power, while fitting an interaction model where there is in fact no interaction leads to negligible losses. The methodology is illustrated on the AneuRisk65 study data.

1. Introduction

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Functional regression models with scalar response and functional covariate have received a considerable amount of attention in the functional data analysis literature. Perhaps one of the most popular functional regression models is the so called functional linear model (FLM), first introduced by Ramsay and Dalzell (1991). A typical FLM with a single functional predictor quantifies the effect of the predictor as an inner product between the functional predictor and an unknown coefficient function; see e.g., Horváth and Kokoszka (2012), Ramsay and Silverman (2005), Ferraty and Vieu (2006), Bongiorno et al. (2014) for general discussions on this type of model. Recently, there has been a lot of interest in functional regression models that relax the linearity assumption used in FLM. For the case of a single functional predictor, current advances in this direction include: purely nonparametric functional regression models (see Delsol, 2013, Ferraty and Vieu, 2006) and functional partially linear models, where the functional covariate is modeled nonparametrically and other scalar or vector valued covariates are modeled parametrically (see e.g., Aneiros-Pérez and Vieu, 2006, Aneiros-Pérez and Vieu, 2008, Lian, 2011, Maity and Huang, 2012, among many others). These models are commonly developed using nonparametric kernel smoothing based-techniques. In the spline smoothing framework, Zhou and Chen (2012) developed spline estimation for a semi-functional linear model, while McLean et al. (2014) and McLean et al. (2015) developed estimation and testing procedures for functional generalized additive models. Non-linear extensions to the usual FLM include single-index models, where instead of modeling the entire functional covariate nonparametrically, one models a linear index (defined by the inner product of the function with an unknown coefficient function) via an unknown smooth function; see e.g., James and Silverman (2005), Amato et al. (2006), Chen et al. (2011), Ferraty et al. (2013) and references therein. A kernel machine regression based approach to fit a linear functional regression model was proposed by Zhao et al.

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(2015). Recently, Kudraszow and Vieu (2013) developed kNN based estimation procedure for nonparametric functional regression models and provided uniform consistency results.

Applications involving two or more functional covariates are becoming increasingly popular. There are several extensions of the simple FLM that incorporate multiple functional predictors: (1) such as generalized functional linear models (James, 2002) for exponential family response variables, (2) penalized functional regression (Goldsmith et al., 2011), (3) group lasso based variable selection for functional linear models (Gertheiss et al., 2013), (4) linear functional additive models for time series prediction (Goia, 2012), among many others. Extensions beyond the linear relationship include: functional partially linear models, where some of the functional covariates are modeled nonparametrically while the rest of the covariates are modeled linearly, see for example Aneiros-Pérez and Vieu (2015) and Lian (2011), among others. Fully nonparametric functional regression models were recently developed for both continuous and general response variables in Muller and Stadtmuller (2005) and Ferraty and Vieu (2009), respectively, where each of the functional predictors is modeled using smooth nonparametric functionals. These articles also include development of functional index models with multiple functional predictors. Recently, Goia and Vieu (2013) proposed a partitioned functional single-index model where the domain of functional covariate is partitioned into several smaller intervals and separate indices are formed for each interval, and the indices are modeled nonparametrically in an additive fashion. Multivariate functional non-parametric models and additive functional non-parametric models are developed by Aneiros-Pérez and Vieu (2006). There are several resources (such as Bongiorno et al., 2014, Ferraty and Vieu, 2006, Horváth and Kokoszka, 2012, Ramsay and Silverman, 2005) that provide extensive discussion on various types of functional regression models; we refer the readers to these resources for further background.

While there is a significant amount of literature available on functional regression with multiple predictors, a common assumption made by all the above mentioned models is that the effects of the functional predictors are additive, that is only the main effects of the individual functional covariates enter the regression model. Thus any interaction between the functional covariates is not taken into account. In general, ignoring such interaction terms may lead to inaccurate and biased estimation of the model parameters which in turn lead to incorrect conclusions. Therefore, development of a functional regression model is needed where one can accommodate both multiple functional predictors as well as interactions among them. In this article, we develop a functional linear interaction model, as well as a penalized spline based estimation procedure for the interaction effect and individual main effects of the functional covariates.

The model we consider is described as follows. Suppose for $i = 1, \ldots, n$, we observe a scalar response Y_i, and independent real-valued, zero-mean, and square integrable random functions $X_{1i}(\cdot)$ and $X_{2i}(\cdot)$ observed without noise, on dense grids. We consider the model

$$E[Y_i|X_{1i}, X_{2i}] = \alpha + \int X_{1i}(s)\beta_1(s)ds + \int X_{2i}(t)\beta_2(t)dt + \int\int X_{1i}(s)X_{2i}(t)\gamma(s,t)dsdt, \quad (1)$$

where α is the overall mean, $\beta_1(\cdot)$ and $\beta_2(\cdot)$ are real-valued functions defined on τ_1 and τ_2 respectively, and $\gamma(\cdot,\cdot)$ is a real valued bi-variate function defined on $\tau_1 \times \tau_2$. The unknown functions β_1 and β_2 capture the main effects of the functional covariates, while γ captures the interaction effect. To gain some insight, consider the particular case $\beta_1(\cdot) \equiv \beta_{01}$, $\beta_2(\cdot) \equiv \beta_{02}$, $\gamma(\cdot,\cdot) \equiv \gamma_0$, for scalars β_{01}, β_{02}, and γ_0. This case reduces to the common two-way interaction model, with covariates $X_{ji} = \int X_{ji}(s)\, ds$, which act as sufficient summaries, $X_{ji},\ j = 1, 2$. Thus the proposed model is an extension of the common two-way interaction model from scalar covariates to functional covariates. The denseness of the sampling design and the noise free assumption are made for simplicity and will be relaxed in later sections.

Recently, Yang et al. (2013) introduced a class of functional polynomial regression models of which model (1) is a special case; they showed that accounting for a functional interaction effect between depth spectrograms and temperature time series improved prediction of sturgeon spawning rates in the Lower Missouri river. The proposed methodology relies on an orthonormal basis decomposition of the functional covariates and parameter functions, combined with stochastic search variable selection in a fully Bayesian framework. Their approach requires full prior specification of several parameters, along with implementation of an MCMC algorithm for model fitting.

The main contribution of this article is a novel approach for estimation, inference and prediction in a parametric functional linear model that incorporates a two-way interaction. We consider a frequentist view and model the unknown functions using pre-determined spline bases and control their smoothness with quadratic penalization. The inclusion of an interaction term between the functional predictors involves additional computational and modeling challenges. A tensor product basis is used to model the interaction surface; such a choice is particularly attractive as it can automatically handle predictors that are on different scales, allows for flexible smoothing in separate directions of the interaction contour, and easily extends to higher dimensions; see de Boor (1978) for important early work, see also Eilers and Marx (2005). The main advantage of our approach is that it can be implemented with readily available software, that accommodates (1) responses from any exponential family, (2) functional covariates observed with error, or on a sparse or dense grid, and (3) produces p-values for individual model components, which include the interaction term. The paper also includes a numerical comparison between the additive and interaction functional models involving scalar response. Our findings can be summarized as follows. When the true model contains an interaction between the functional covariates, as specified in (1), then fitting a simpler additive model (Goldsmith et al., 2011) leads to biased estimates and low prediction performance compared to fitting a functional interaction model. When the true model contains no interaction effect, then with sufficient sample size, fitting the more complex functional interaction model does not harm the estimation, inference or prediction performance.

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