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Modeling process asymmetries with Laplace moving average

Nicolas Raillard [a,*], Marc Prevosto [a], Pierre Ailliot [b]

[a] Laboratoire Comportement des Structures en Mer, IFREMER, France

[b] Laboratoire de Mathématiques de Bretagne Atlantique, Université de Bretagne Occidentale, Brest, France

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ABSTRACT

Many records in environmental science exhibit asymmetries: for example in shallow water and with variable bathymetry, the sea wave time series shows front–back asymmetries and different shapes for crests and troughs. In such situation, numerical models are available but their computational cost and complexity are high. A stochastic process aimed at modeling such asymmetries has recently been proposed, the Laplace moving average process, which consists in applying a linear filter on a non-Gaussian noise built using the generalized Laplace distribution. The objective is to propose a new non-parametric estimator for the kernel involved in the definition of this process. Results based on a comprehensive numerical study will be shown in order to evaluate the performances of the proposed method.

1. Introduction

Marine coastal systems are subject to loadings due to sea waves, and long time series of the loadings are often needed to carry out studies of the performances of the system or to assess its extremal behavior. During its lifetime, this system is likely to encounter various sea states, and thus one needs to be able to simulate long and numerous series with realistic characteristics. Many such systems are located near the shore, in shallow water and with variable bathymetry. In this context, the waves are known to be non-linear, and show high asymmetries. Two kinds of asymmetries can arise in this context: the top–bottom (vertical) asymmetry and the front–back (horizontal) asymmetry. The first one describes the different behavior of troughs compared to crests with the peaks being generally sharper compared to the bottom. The second kind of asymmetry is linked to the time irreversibility: for example, the front steepness of crests is higher than the back one. Many other situations in natural and social sciences lead to similar asymmetries.

The aforementioned criteria are known to be impossible to reproduce with Gaussian processes, and models with such asymmetries must be developed. Recently, such a model has been proposed (Podgórski and Wegener, 2010; Aberg and Podgórski, 2011), along with a description of some of its properties (Galtier et al., 2010; Galtier, 2011) and an estimation procedure of some of its characteristics (Podgórski and Wegener, 2011). The model is a linear filter of a non-Gaussian noise built using the generalized Laplace distribution. The goal of the study is thus to propose a new method for estimating the kernel, which is an unknown function that rules the behavior of the process, and then to study the behavior of this estimator both on simulated and real data.

The paper is organized as follows. Section 1 introduces the model and some of its characteristics, and in particular the high-order spectrum properties that are used in the estimation procedure which is discussed in Section 2. In Section 3, we present a simulation study to assess the performances of the new estimation procedure. An application on a real dataset is carried out in Section 4. Conclusions and key findings are given in Section 5.

* Corresponding author.
E-mail addresses: nicolas.raillard@gmail.com (N. Raillard), marc.prevosto@ifremer.fr (M. Prevosto), pierre.ailliot@univ-brest.fr (P. Ailliot).

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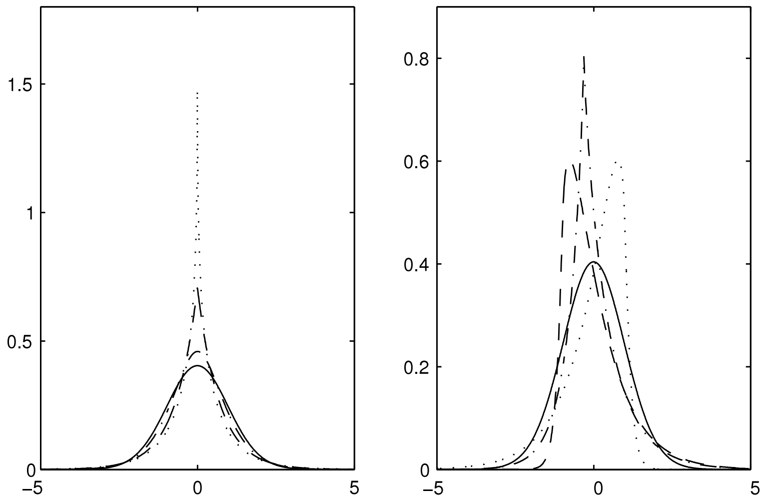

Fig. 1. Densities of GAL distributions.

2. Description of the LMA model

2.1. Model construction

The construction of the process is based on a non-Gaussian noise, itself based on a non-Gaussian distribution, intended to be flexible and capable to handle heavy tails and distributional asymmetries, the *Generalized Asymmetric Laplace distribution*.

Definition 1 (*Generalized Asymmetric Laplace Distribution*)**.** The following characteristic function defines a distribution called 'Generalized Asymmetric Laplace distribution' ($\mathcal{GAL}$):

$$\Phi(u) = e^{i\delta u}\left(1 - i\mu u + \frac{\sigma^2 u^2}{2}\right)^{-1/\nu},$$

where $\delta,\ \mu \in \mathbb{R}$, $\nu > 0$ and $\sigma > 0$. The cases $\nu = 1$ and $\mu = 0$ are referred to as the asymmetric Laplace distribution and the generalized symmetric Laplace, respectively.

This distribution has finite moments of any order and in particular if $Y \sim \mathcal{GAL}(\delta, \mu, \sigma, \nu)$, then

$$\mathbb{E}Y = \frac{\mu}{\nu} + \delta \qquad \mathbb{V}Y = \frac{\mu^2 + \sigma^2}{\nu}$$

$$s(Y) = \mu\sqrt{\nu}\frac{2\mu^2 + 3\sigma^2}{(\mu^2 + \sigma^2)^{3/2}} \qquad \kappa_e(Y) = 3\nu\left(2 - \frac{\sigma^4}{(\mu^2 + \sigma^2)^2}\right),$$

where s denotes the skewness and κ_e the excess kurtosis. This distribution shows great flexibility, with various shapes for the density function as it can be seen in Fig. 1. This figure shows 8 densities all such that the mean is equal to 0 and variance to 1. The left plot contains symmetric distributions, while the right plot shows asymmetric densities.

Thanks to the infinite divisibility property of the $\mathcal{GAL}$ distribution, one can construct also a Levy process, with stationary and independent increments from a $\mathcal{GAL}$, called a Laplace motion (see Podgórski and Wegener, 2011):

Definition 2 (*Laplace Motion*)**.** A Laplace motion Γ is defined by the following conditions:

- it starts at the origin (i.e. $\Gamma(0) = 0$);
- its increments are stationary and independent;
- the increments by the time unit ν have a zero mean asymmetric Laplace distribution.

This process is also referred as the Variance-Gamma process, see Kotz et al. (2001).

Once we have such a process constructed, it becomes possible to define the process that will be intensively used in the sequel, namely a convolution of a Laplace motion with some function. This leads to a stationary linear process which is linear but non-Gaussian, with the ability to produce asymmetries, see Podgórski and Wegener (2010) for more details.