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Multivariate distributions with proportional reversed hazard marginals

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ABSTRACT

Several univariate proportional reversed hazard models have been proposed in the literature. Recently, Kundu and Gupta (2010) proposed a class of bivariate models with proportional reversed hazard marginals. It is observed that the proposed bivariate proportional reversed hazard models have a singular component. In this paper we introduce the multivariate proportional reversed hazard models along the same manner. Moreover, it is observed that the proposed multivariate proportional reversed hazard model can be obtained from the Marshall–Olkin copula. The multivariate proportional reversed hazard models also have a singular component, and their marginals have proportional reversed hazard distributions. The multivariate ageing and the dependence properties are discussed in details. We further provide some dependence measure specifically for the bivariate case. The maximum likelihood estimators of the unknown parameters cannot be expressed in explicit forms. We propose to use the EM algorithm to compute the maximum likelihood estimators. One trivariate data set has been analysed for illustrative purposes.

1. Introduction

If X is an absolutely continuous positive random variable with the probability density function (PDF) $g(\cdot)$ and the cumulative distribution function $G(\cdot)$, then the hazard function of X is defined as

$$h(t) = \frac{g(t)}{1 - G(t)}; \quad t \geq 0.$$

The hazard function plays a very important role in the reliability and survival analysis. Extensive work on different aspects of hazard function has been found in the statistical literature, see for example Meeker and Escobar (1998).

Recently, proportional reversed hazard model has received considerable attention since it was introduced by Block et al. (1998). If X is an absolutely continuous positive random variable as defined above, then the reversed hazard function of X is defined by

$$r(t) = \frac{g(t)}{G(t)}; \quad t \geq 0.$$

Similar to the hazard function, the reversed hazard function also uniquely characterize a distribution function. The reversed hazard function has been used quite extensively in forensic studies and some related areas. Interested readers may look at the original article of Block et al. (1998) in this respect.

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The class of proportional reversed hazard models can be described as follows. If $F_0(\cdot)$ is any distribution function, then define the class of distribution functions $F(\cdot;\alpha)$ for $\alpha > 0$ as

$$F(t;\alpha) = (F_0(t))^{\alpha}.$$

It can be easily seen that $F(\cdot;\alpha)$ is a proper distribution. From the definition of the proportional reversed hazard function, it is immediate that if $F_0(\cdot)$ has a reversed hazard function $r_0(\cdot)$, then $F(\cdot;\alpha)$ has the proportional reversed hazard function $\alpha r_0(\cdot)$. Recently several proportional reversed hazard models have been introduced by several authors, and their properties have been investigated, see for example Crescenzo (2000), Kundu and Gupta (2004), Gupta and Gupta (2007), Gupta and Kundu (1999, 2007), Sarhan and Kundu (2009) and the references cited therein.

Kundu and Gupta (2010) recently introduced a bivariate distribution with proportional reversed hazard marginals. It has several interesting properties, and it has been used quite successfully to analyse bivariate lifetime data. The main aim of this paper is to introduce multivariate (p-dimensional) distributions with proportional reversed hazard marginals. It has been done using the same maximization process from $p + 1$ independent proportional reversed hazard models. It introduces positive dependence among the variables. The proposed multivariate proportional reversed hazard model can be obtained from the Marshall–Olkin (MO) copula also, using the proportional reversed hazard model as the marginals. Using the copula properties, several dependence measures like Kendall's τ, Spearman's ρ can be computed specifically for the bivariate proportional reversed hazards distribution.

It is observed that for $q < p$ dimensional subset of the p-variate proportional reversed hazards distribution is a q-variate proportional reversed hazards distribution. The cumulative distribution function of the q-variate proportional reversed hazards distribution can be written in a very convenient form. The decomposition of the absolutely continuous part and the singular part is clearly unique. We provide the joint probability density function of the absolute continuous part explicitly. We discuss some distributional, ageing and dependence properties for the proposed p-variate distribution.

It may be mentioned that the importance of the ageing and dependence notions has been well established in the statistical literature, see for example Lai and Xie (2006). In many reliability and survival analysis applications it has been observed that the components are often positively dependent in some stochastic sense. Hence the derivation of ageing and dependence properties for any multivariate distribution has its own importance. Similarly, the extreme order statistics, the minimum and maximum play a great role in several statistical applications, particularly, where the components have some dependence. For example, the minimum and maximum order statistics play important roles in the competing risks model, and the complementary risks model, respectively. So the distributions of both extreme order statistics for the proposed multivariate distributions and some stochastic ageing results are studied in this paper.

It is observed that the maximum likelihood estimators (MLEs) of the unknown parameters cannot be obtained in explicit form, as expected. Non-linear optimization problem needs to be solved to compute the MLEs. We propose to use the EM algorithm to compute the MLEs, and we provide the implementation details for several multivariate proportional models. Finally, we analyse one real data set for illustrative purposes.

Rest of the paper is organized as follows. In Section 2, we briefly discuss about the different dependence concept, some basic copula properties and provide different examples of proportional reversed hazards models which are available in the literature. In Section 3, we introduce the multivariate proportional reversed hazards models. Different dependence and ageing properties are discussed in Section 4. In Section 5, we provide different dependence measures for bivariate proportional reversed hazards models. In Section 6, we apply the EM algorithm. The analysis of a data set has been presented in Section 7, and finally we conclude the paper in Section 8.

2. Preliminaries

2.1. Dependence and stochastic order

Several notions of positive or negative dependence for a multivariate distribution, of varying degree of strengths, are available in the literature, see for example Boland et al. (1996), Colangelo et al. (2005, 2008) and see the references cited therein.

A random vector $\boldsymbol{X}$ is said to be positively lower orthant dependent (PLOD) if the joint cumulative distribution function of $\boldsymbol{X}$ satisfies the following property;

$$F_{\boldsymbol{X}}(x_1,\ldots,x_p) \geq \prod_{i=1}^{p} F_i(x_i), \quad \forall \boldsymbol{x} = (x_1,\ldots,x_p), \tag{1}$$

here F_i's for $i = 1,\ldots,p$, are the marginal distribution functions. Further we will be using the following notation. For $\boldsymbol{x} \in \mathbf{R}^p$, a phrase such as 'non-decreasing in $\boldsymbol{x}$', means non-decreasing in each component x_i, for $i = 1,\ldots,p$. If A is any subset of $\{1,\ldots,p\}$, then $\boldsymbol{X}_A$ denote the vectors, $(X_i|i \in A)$, similarly, $\boldsymbol{x}_A$ is also defined. The following definition are from Brindley and Thompson (1972), see also Joe (1997).

A p-dimensional random vector $\boldsymbol{X}$ is said to be left tail decreasing (LTD), if

$$P(\boldsymbol{X}_{A_2} \leq \boldsymbol{x}_{A_2}|\boldsymbol{X}_{A_1} \leq \boldsymbol{x}_{A_1}) \tag{2}$$