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Statistical inference for population quantiles and variance in judgment post-stratified samples

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HIGHLIGHTS

- A quantile inference is developed based on a judgment post-stratified (JPS) sample.
- A variance estimator is constructed for population variance based on a JPS sample.
- The procedures have higher efficiencies than the ones in a simple random sample.
- The procedures have slightly lower efficiencies than the ones in a ranked-set sample.
- Inference in a JPS sample is asymptotically equal to the one in a ranked-set sample.

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ABSTRACT

A judgment post-stratified (JPS) sample is used in order to develop statistical inference for population quantiles and variance. For the pth order of the population quantile, a test is constructed, an estimator is developed, and a distribution-free confidence interval is provided. An unbiased estimator for the population variance is also derived. For finite sample sizes, it is shown that the proposed inferential procedures for quantiles are more efficient than corresponding simple random sampling (SRS) procedures, but less efficient than corresponding ranked set sampling (RSS) procedures. The variance estimator is less efficient, as efficient as, or more efficient than a simple random sample variance estimator for small, moderately small, and large sample sizes, respectively. Furthermore, it is shown that JPS sample quantile estimators and tests are asymptotically equivalent to RSS estimators and tests in their efficiency comparison.

1. Introduction

Main motivation behind judgment post-stratification (JPS) and ranked set sampling (RSS) designs is to use a collection of a few number of experimental units without measurement to create homogeneous groups of measured observations. In a ranked set sample, for each measured observation, researcher selects a set of H experimental units at random from a population of interest. These units are assigned ranks 1 through H. Ranks are assigned without measurement based on either some concomitant variables or subjective opinion of the researcher on the size of the units. Out of these H units, only one unit is measured, yielding a single observation, $X_{[r_i]i}$, where r_i is the rank of the measured unit. The remaining $H-1$ units in the set provide a context for the measured observation, determining its relative standing in the set through its rank. This process is repeated n times to construct a ranked set sample of size n, $X_{[r_i]i}$, $i = 1, \ldots, n$. The square brackets are used to indicate that ranking of the units in the sets may be in error. If the ranking process is error-free, the square brackets are

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replaced with the round ones. In this case, judgment ranked order statistics become usual order statistics form a simple random sample of size H. The judgment ranks in a ranked set sample act like strata in stratified sampling design by putting similar observations in the same judgment group. Hence, efficiency improvement in an RSS design can be anticipated from the general theory or a stratified sampling design.

A JPS sample, proposed by MacEachern et al. (2004), starts with a simple random sample and uses additional experimental units to create post-strata among already measured observations. The construction of a JPS sample requires selecting a sample size n and set size H. One then selects a simple random sample, X_i; $i = 1, \ldots, n$, of size n and measures all of them. For the ith measured unit, a JPS sample needs $H - 1$ additional units to form a set of size H. The units in this set are ranked from smallest to largest without a measurement, and the rank of the measured unit, on which X_i is already measured, is recorded. The full JPS data then consist of n measured values and n ranks associated with these measured values, (X_i, R_i), $i = 1, \ldots, n$, where R_i is the rank of X_i. Note that the capital letter R_i is used to denote that the ranks in a JPS sample are random variables.

It is clear from the description of RSS and JPS samples, there are two major differences between these two designs. The first difference is in the way that the ranking process is applied to the units in each set. In an RSS design, ranking is performed prior to measurement of a unit in a set. On the other hand, ranking is performed after measurement in a JPS design. As a result, the assigned ranks in a ranked set sample design become a part of the measured observations. They cannot be separated from the observed values. Hence, an RSS sample must be analyzed with an appropriate statistical procedure developed specifically for an RSS design. In a JPS sample, since the ranks are assigned after the fact that a simple random sample (SRS) has been collected, it can be analyzed as an SRS by ignoring the ranking information.

Another significant difference between a JPS and RSS sample is due to distributional properties of the sample size vector of the judgment rank classes. Let n_h, $h = 1, \ldots, H$, be the number of measured observations in the hth judgment class in an RSS sample. The sample size vector of judgment classes in an RSS sample, $(n_1, \ldots, n_H)$, is a deterministic vector and must be determined prior to the construction of the sample while it is a random vector in a JPS sample. Therefore, a JPS sample could be highly unbalanced for small sample sizes and tends to create empty strata. This additional variation in the sample size vector makes a JPS sample less efficient in comparison with an RSS sample. It is then important to investigate the impact of the random sample size vector on a JPS sample.

The JPS sampling design has drawn attention in recent literature. Wang et al. (2008) constructed an estimator for the population mean based on stochastic ordering of judgment class distributions. Stokes et al. (2007) and Wang et al. (2006) constructed several estimators for population mean based on a JPS sample that combines ranking information of multiple rankers. Frey and Ozturk (2011) developed an estimator for the population cumulative distribution function (cdf) that uses a weaker ordering constraint than the stochastic ordering constraint of the judgment class distributions. Ozturk (2012) introduced a sampling scheme that incorporates ranking information from multiple sources to improve the JPS sampling design. For small sample sizes, it is highly possible that the JPS sample may contain empty strata. Wang et al. (2012) introduced an estimator for the population cdf that deals with the empty strata in JPS samples. Frey and Feeman (2012) showed that standard JPS estimator of a population mean is inadmissible and constructed an optimal estimator within a class of unbiased estimators. Ozturk (2013a) developed a class of estimators for population moments and median in an infinite population setting. Ozturk (2013b) constructed estimators for the population mean and total in a finite populations setting. Most recently, Dastbaravarde et al. (in press) provided a theoretical framework to show that JPS moment estimators are less efficient than RSS moment estimators for finite sample sizes.

The main contribution of this research is to use a JPS sample to draw distribution-free statistical inference for the population quantile of order p and the population variance. Section 2 introduces a quantile loss function to estimate the pth quantile of a population. The estimating equation of this quantile function is used to develop a two-sided distribution-free test for the population quantile. The test statistic is calibrated to minimize the impact of possible ranking error in the ranking process. Section 3 constructs a distribution-free confidence interval. Section 4 introduces another estimator for the population quantile that minimizes the quantile loss function under the stochastic ordering constraint of the judgment class cumulative distribution functions (cdf). Section 5 introduces an unbiased estimator for the population variance. Section 6 provides empirical results to evaluate the finite sample properties of the estimators, tests and confidence intervals. Finally, Section 7 provides a concluding remark.

2. Quantile estimator and test

Let (X_i, R_i); $i = 1, \ldots, n$, be a JPS sample from a distribution F having absolutely continues density function f. Note that in a JPS sample the ranks R_i; $i = 1, \ldots, n$, are random variables having discrete uniform distributions on the set of integers $\{1, \ldots, H\}$. Let $\boldsymbol{R}$ be the rank vector in a JPS sample, $\boldsymbol{R}^\top = (R_1, \ldots, R_n)$, and let $I_{i,h}$ be one if X_i has rank h $(R_i = h)$ and zero otherwise. The number of observations having rank h is then denoted by $N_h = \sum_{i=1}^{n} I_{i,h}$. The sample size vector $\boldsymbol{N}^\top = (N_1, \ldots, N_H)$ has a multinomial distribution with parameters $(n, 1/H, \ldots, 1/H)$ with $n = \sum_{h=1}^{H} N_h$. Since $\boldsymbol{N}$ is a multinomial random vector, there is a positive probability (depending on the sample size n) that some of the N_h could be zero. Let I_h be the indicator function for non-empty judgment classes, i.e. I_h is one if $N_h > 0$ and zero otherwise. Let d_n be the number of nonzero N_h in a JPS sample, i.e. $d_n = \sum_{h=1}^{H} I_h$. It is important to recognize that d_n is a random variable.